

# Introduction to Graphs

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- State the Handshaking Theorem and explain why it works (informally).

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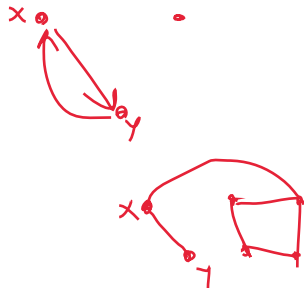
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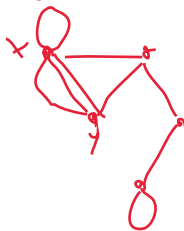




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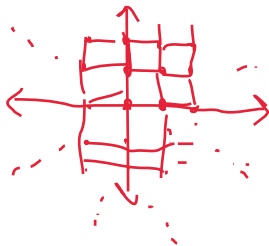
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no mult. edges, no self-loops



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## Warning

Default assumptions: simple, undirected, at least one vertex, finite

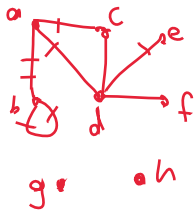
"graph"

$$1 \leq |V| < \infty$$

# The sum of vertex degrees counts every edge twice.

## Definition

The **degree** of a vertex in an undirected graph is the number of edges incident to it.



$$\deg(a) = 3$$

$$\deg(b) = 3$$

$$\deg(e) = 1$$

$$\deg(h) = 0$$

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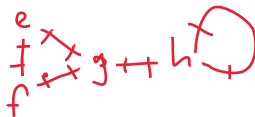
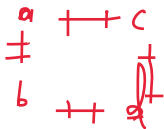
## Theorem (Handshaking Theorem)

Let  $G = (V, E)$  be an undirected graph. Then

$$\sum_{k \in \{1, 2, \dots, n\}} \frac{1}{2^k} \quad \sum_{v \in V} \deg(v) = 2|E|.$$

*set of edges*

$$|E| = \frac{\sum_{v \in V} \deg(v)}{2}$$



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