Onto Functions

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Learning Objectives

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- Prove a given function is (not) onto.

When is a function onto?

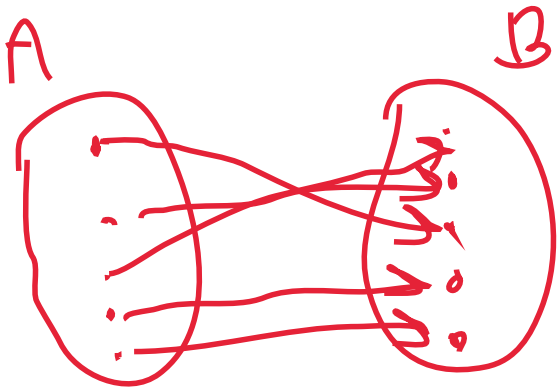
Definition

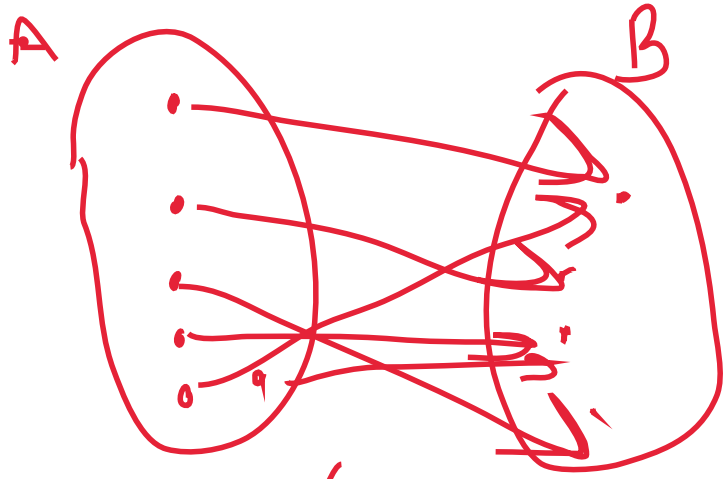
A function $f : A \to B$ is ***onto*** if its image is its entire co-domain, that is, $f(A) = B$.

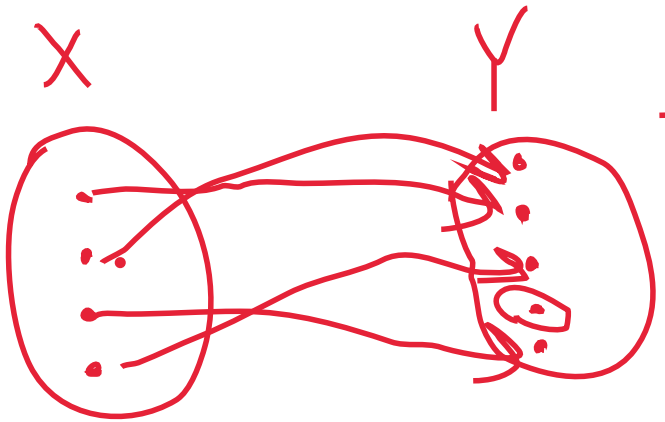
↓ surjective

$f(x) = y$

$\forall y \in B, \exists x \in A, f(x) = y.$







$f: \mathbb{Z} \to \mathbb{Z}$

$f(n) = 0.$

preimage

If $|X| < |Y|$, then $f: X \to Y$ cannot be onto.

Proving Onto

Proving Onto

$$\forall y \in B, \exists x \in A, f(x) = y$$

1. Pick arbitrary $y \in B$.
2. Give example (construct) $x \in A$ s.t. $f(x) = y$.

Proving Onto

$\forall y \in B, \exists x \in A, f(x) = y$

Example

$f : \mathbb{Z} \to \mathbb{Z}, f(n) = \left\lfloor \frac{3n-2}{4} \right\rfloor$

Proof: Let $y \in \mathbb{Z}$ be arbitrary.

Choose $n = \left\lceil \frac{4y+2}{3} \right\rceil$.

$\vdots$

$\left\lfloor \frac{3n-2}{4} \right\rfloor = y$

$\frac{3n-2}{4} = y$

$n = \left\lceil \frac{4y+2}{3} \right\rceil$

So $f(n) = y$, and we conclude f is onto. $\square$

Proving NOT Onto

Proving NOT Onto

onto

$$\neg(\forall y \in B, (\exists x \in A, (f(x) = y))) \equiv \exists y \in B, \forall x \in A, f(x) \neq y$$

Proving NOT Onto

$\neg(\forall y \in B, \exists x \in A, f(x) = y) \equiv \exists y \in B, \forall x \in A, f(x) \neq y$

Example

$g : \mathbb{Z} \to \mathbb{Z}, g(n) = 5n - 4$

$g(\mathbb{Z}) = [1]_5$ $[0], [2], [3], [4]$

$\forall x, y \in \mathbb{R}, \cdots$ $\forall x \in A, \exists y \in B, P(x,y)$

$\exists y \in B, \forall x \in A, P(x,y)$

Proof: Let $y = 0$.

Let $n \in \mathbb{Z}$ be arb.

Then $g(n) = 5n - 4 \equiv 1 \pmod 5$

$\not\equiv 0 \pmod 5$,

so $g(n) \neq 0$.

Hence g is not onto. $\square$

Recap: Learning Objectives

By the end of this lesson, you will be able to:

- Define and recognize onto functions.
- Prove a given function is (not) onto.