Introduction to Functions

Ian Ludden

Learning Objectives

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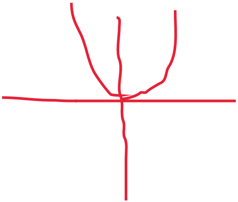
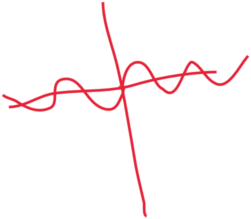
- Count the number of possible functions from A to B.
- Determine whether a given formula or diagram defines a function.

Function Formalities

$f(x) = x^2$



$g(x) = \sin(x)$



Function Formalities

Definition

A ***function*** $f : A \to B$ is a mapping of each input in A (the ***domain***) to exactly one element in B (the ***co-domain***).

$f(x) = x^2$

$g(x) = \sin(x)$

$f : \mathbb{R} \to \mathbb{R}$

$g : \mathbb{R} \to \mathbb{R}$

$g : \mathbb{R} \to [-1, 1]$

~~range~~

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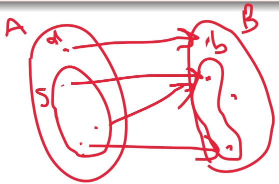
Definition

For a given $x \in A$, $f(x)$ is the ***image*** of x. This extends to any subset $S \subseteq A$: $f(S) = \{b \in B : \exists a \in S f(a) = b\}$.

$b \in B$



Definition

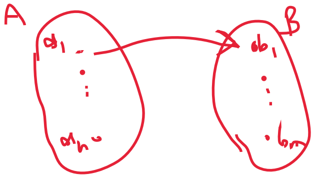
The ***identity*** function for a set A, denoted id_A, is $f : A \to A$, $f(a) = a$.

$A = \mathbb{Z}$ $\qquad$ $\mathrm{id}_A(n) = n.$

$A = \{0, 1, 2, 3\}$ $\qquad$ $f(n) =$ remainder when $n + 400$ is div. by 4

Counting Functions

Consider sets A and B with $|A| = n$ and $|B| = m$. How many distinct functions are possible from A to B?



$m \cdot m \cdot \ldots \cdot m = m^n$.

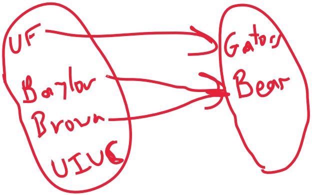
choices for $f(a_1)$

$g: B \to A$?

n^m such fn.s

Will it ~~blend~~ function?

- Universities to their mascots



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- $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = \lfloor x \rfloor$

$[0, 1)$

Will it ~~blend~~ function?

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- $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = \lfloor x \rfloor$
- $g : \mathbb{Z} \to \mathbb{Z}$, $g(n) = \{m \in \mathbb{Z} : m \mid n\}$

$f(6) = \{1, 2, 3,$

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- $g : \mathbb{Z} \to \mathbb{Z}$, $g(n) = \{m \in \mathbb{Z} : m \mid n\}$
- $g : P \to P$, $g(n) = \{m \in P : m \mid n\}$

$\{5\}$

$= n$

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