Introduction to Set Theory

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Learning Objectives

By the end of this lesson, you will be able to:

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- Recall basic set theoretic definitions and notation.

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By the end of this lesson, you will be able to:

- Recall basic set theoretic definitions and notation.
- Compute basic operations on concrete sets.

Definitions via Examples

Definition

A ***set*** is an unordered collection of objects.

Definitions via Examples

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Examples

- $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{C}$

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- $\mathbb{Z}$
- $\{\text{apple}, \text{banana}, \text{orange}\}$

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- $\mathbb{Z}$
- $\{\text{apple}, \text{banana}, \text{orange}\}$
- $\{a \in \mathbb{N} \mid a \leq 10\}$ set builder notation

$\{0, 1, 2, \ldots, 10\}$

$|$ or $:$ "such that"

Definitions via Examples

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- $\mathbb{Z}$
- $\{\text{apple}, \text{banana}, \text{orange}\}$
- $\{a \in \mathbb{N} \mid a \leq 10\}$
- $\cancel{\{\}} = \emptyset$

$\emptyset$ ϕ

$\frac{3}{5} \cdot \frac{45}{3} = \frac{9}{1} = 9$

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- The equivalence class of 4 mod 7:

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  - $\{\dots, -10, -3, 4, 11, 18, \dots\}$

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  - $\{m \in \mathbb{Z} : m \equiv 4 \pmod{7}\}$

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  - $\{m \in \mathbb{Z} : m \equiv 4 \pmod{7}\}$
  - $\{m \in \mathbb{Z} : 7 \mid (m - 4)\}$

$\{m \in \mathbb{Z}:$ 7 divides $(m-4)\}$

Definition

The ***cardinality*** of a set S, denoted $|S|$, is the number of distinct objects it contains.

$|\mathbb{Z}| = \infty$

$|\{apple, banana, orange\}| = 3$

$|\{a \in \mathbb{N} \mid a \leq 10\}| = 11$ $\quad (0, 1, 2, \ldots, 10)$

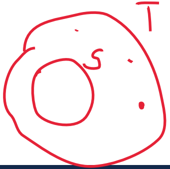
Definition

The ***cardinality*** of a set S, denoted $|S|$, is the number of distinct objects it contains.

Definition

Given sets S and T, we call S a ***subset*** of T (denoted $S \subseteq T$) if every element in S is also an element of T. We call S a ***proper*** subset of T (denoted $S \subset T$) if T has at least one element that S doesn't.

$S \subseteq T$



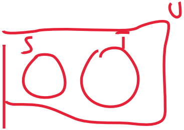
$S \subset T$.

Set Operations

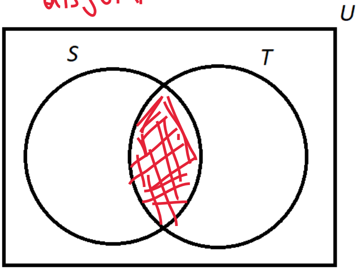
Let S and T be sets in universe U.

- Intersection:
  $S \cap T := \{s : s \in S \wedge s \in T\}$

$S \cap T = \emptyset \iff S, T$ disjoint



"disjoint"



Set Operations

Let S and T be sets in universe U.

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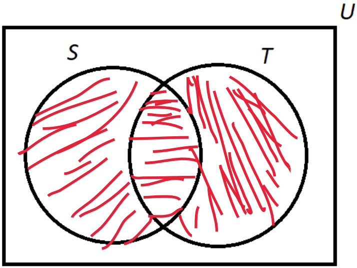
  $S \cap T := \{s : s \in S \overset{\text{and}}{\cancel{\land}} s \in T\}$
- Union:
  $S \cup T := \{s^{\in U} : s \in S \overset{\text{or}}{\cancel{\lor}} s \in T\}$



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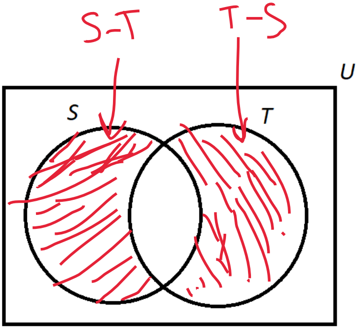
- Difference:
  $S \setminus T := S - T :=$
  $\{s \in S : s \notin T\}$



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- Complement:
  $\overline{S} := \{s \in U : s \notin S\}$

  $:= U - S$

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- Complement:
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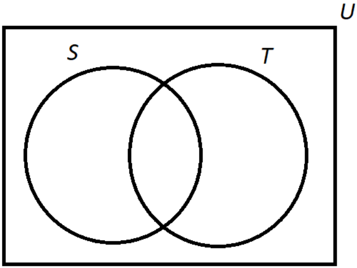
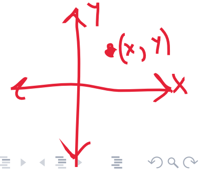
- Cartesian product:
  $S \times T :=$
  $\{(s, t) : s \in S \wedge t \in T\}$



$S = T = \mathbb{R}$

$S \times T = \mathbb{R} \times \mathbb{R} = \mathbb{R}^2$



Computing Set Operations

$U := \mathbb{Z}$

$A := \{a \in \mathbb{Z} : a \geq 4 \text{ or } a < 0\}$ $= \{4, 5, 6, 7, \ldots\} \cup \{-1, -2, -3, \ldots\}$

$B := \{b \in \mathbb{Z} : b \text{ is odd and } |b| < 6\}$ $= \{-5, -3, -1, 1, 3, 5\}$

$A \cap B = \{5, -1, -3, -5\}$ $\quad |A \cap B| = 4$

$A \cup B = \{n \in \mathbb{Z} : (n \geq 4 \text{ or } n < 0) \text{ or } (n \text{ is odd and } |n| < 6)\}$

$B \setminus A = B - A = \{1, 3\}$

$\overline{A} = \mathbb{Z} - A = \{0, 1, 2, 3\}$

$\overline{A} \times B = \{(0,-5), (0,-3), (0,-1), (0,1), (0,3), (0,5),$
$(1,-5), (1,-3), \ldots, (3,5)\} = \{(m,n) \in \mathbb{Z}^2 : 0 \leq m \leq 3 \text{ and } n \text{ is odd and } |n| < 6\}$

Recap: Learning Objectives

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