Modular Arithmetic

Ian Ludden

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- $[-7] = \{-7, -2, 3, 8, \ldots\} = [3]$

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- $[6]^{173} = [-1]^{173} = [(-1)^{173}] = [-1] = [6]$

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