Number Theory: Prime Numbers

Ian Ludden

lan Ludden Number Theory: Prime Numbers

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• Compute the prime factorization of (small) positive integers.

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- Compute the prime factorization of (small) positive integers.
- State the Fundamental Theorem of Arithmetic.

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- Compute the prime factorization of (small) positive integers.
- State the Fundamental Theorem of Arithmetic.
- Remember that there are infinitely many prime numbers.

Why study prime numbers?

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"Mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers, and we have reason to believe that it is a mystery into which the human mind will never penetrate." – Euler

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Aside: https://projecteuler.net/

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Prime numbers are:

Fascinating

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Prime numbers are:

- Fascinating
- Essential in cryptography

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Prime numbers are:

- Fascinating
- Essential in cryptography
- Give us handy prime factorizations (short way of representing large integers)

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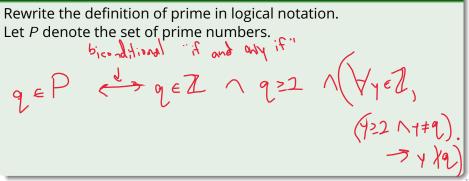
An integer $q \ge 2$ is called *prime* if the only positive factors of q are q and 1. An integer $q \ge 2$ is called *composite* if it is not prime.

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Exercise



What about 0 and 1?

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What about 0 and 1? Neither prime nor composite

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What about 0 and 1? Neither prime nor composite

What about negative integers?

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Prime Factorization

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Examples

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Examples

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$$12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$$

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$$210 = 2 \cdot 3 \cdot 5 \cdot 7 = 3 \cdot 2 \cdot 7 \cdot 5$$

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The *prime factorization* of an integer $q \ge 2$ is the expression of q as the product of one or more prime factors.

Examples

- $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
- $210 = 2 \cdot 3 \cdot 5 \cdot 7$
- $91 = 7 \cdot 13 = 13 \cdot 7$

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The *prime factorization* of an integer $q \ge 2$ is the expression of q as the product of one or more prime factors.

Examples

- $12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$
- $210 = 2 \cdot 3 \cdot 5 \cdot 7$
- $91 = 7 \cdot 13$
- 173 = 173

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Fundamental Theorem of Arithmetic

Theorem (Euclid)

Every integer greater than one has a **unique**^{*} prime factorization.

* Ignoring the order in which we write the factors

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Infinitely Many Primes

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Infinitely Many Primes

Theorem

There are infinitely many prime numbers.

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Theorem

There are infinitely many prime numbers.

Proof sketch
Shppoa not. let
$$P_{1}, P_{2}, P_{3}, \cdots, P_{n}$$

Let $p = P_{1}, P_{2}, P_{3}, \cdots, P_{n} + |$.
 $P_{1} \neq P_{2}, P_{2} \neq P_{3}, \cdots, P_{n} \neq |$.
 $P_{1} \neq P_{2}, P_{2} \neq P_{3}, \cdots, P_{n} \neq P_{n} \Rightarrow P_{n} \Rightarrow$

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Infinitely Many Primes

Theorem

There are infinitely many prime numbers.

Proof sketch

Not a formula for generating primes...

Infinitely Many Primes

Theorem

There are infinitely many prime numbers.

Proof sketch

Not a formula for generating primes. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 + 1 = 510, 511 = 19 \cdot 97 \cdot 277$

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