# Proving Universal Claims

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- Prove universal claims directly.
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- Prove universal claims by splitting into cases.

# Direct proof: The hard/impossible way

Goal: Prove $\forall x \in X, p(x)$.

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Strategy: Individually show the predicate is true for each element in the universe.

# Direct proof: The more reasonable way

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Strategy: Pick an *arbitrary* element in the universe, and show the predicate is true for that element.

# Direct proof: The more reasonable way

Example: Prove $\forall n \in \mathbb{N}, n^2 - 4n + 5 \geq 0$.

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Proof:

# Proof by contrapositive

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Equivalent: Prove $\forall x \in X, \neg q(x) \rightarrow \neg p(x)$.

# Proof by contrapositive

## Example

Prove: $\forall x, y \in \mathbb{R}, x + y \geq 0 \rightarrow (x \geq 0) \vee (y \geq 0)$.

# Proof by cases

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Equivalent:

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   *(It's ok if your sets overlap.)*

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Equivalent:

1. Find sets $X_1, X_2, \ldots, X_k$ such that $X = X_1 \cup X_2 \cup \ldots \cup X_k$.
   *(It's ok if your sets overlap.)*
2. Prove each case, that is, for $i = 1$ to $k$, prove $\forall x \in X_i, p(x)$.

# Proof by cases

## Example 1

Claim: For every integer $n$, $n^3 \geq 0$ or $n \geq 2n$.

# Proof by cases

## Example 2

Claim: Every set of four people includes a trio of friends or a pair of strangers.

# Recap: Learning Objectives

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