

Introduction to Proofs

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Learning Objectives

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- Disprove a universal claim with a concrete counterexample.

Negating statements with quantifiers

$$\neg [\forall x p(x)] \equiv \exists x \neg p(x)$$

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Example

Everybody loves Raymond.

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Example

There is an integer whose value is three less than that of its square.

Proving existential claims

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There is an integer whose value is **six** less than that of its square.

Disproving universal claims

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- 1 Negate the statement to get an existential claim.

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- 2 Prove the existential claim with a concrete (counter)example.

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Example

Every prime number can be expressed as the sum of exactly two prime numbers (not necessarily distinct).

Recap: Learning Objectives

By the end of this lesson, you will be able to:

- Negate statements with quantifiers.
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