Logic: Laws and Transformations

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Learning Objectives

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- Construct the negation, converse, and contrapositive of simple statements.

Translating English into logical shorthand

Example 1:

- If s is a square, then s is a rectangle.

q r

$q \rightarrow r$

Translating English into logical shorthand

Example 2:

- x is less than six, and x is prime or x is not equal to 1.

p q $\neg r$

$p \wedge (q \vee \neg r)$

Translating English into logical shorthand

Example 3:

- For every natural number n, n is even or n is odd.

$\forall n \in \mathbb{N}, p(n) \vee q(n).$

Translating English into logical shorthand

Example 4:

- There exists an integer y such that y squared is 3.

$q(y)$

$\exists y \in \mathbb{Z}, q(y)$

$\exists y \in \mathbb{Z}, y^2 = 3.$

Basic laws of logic

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- Double negation: $\neg(\neg p) \equiv p$ $\neg(\neg\neg p) \equiv \neg p$

Basic laws of logic

- Double negation:
- Distributive laws:

p, q, r

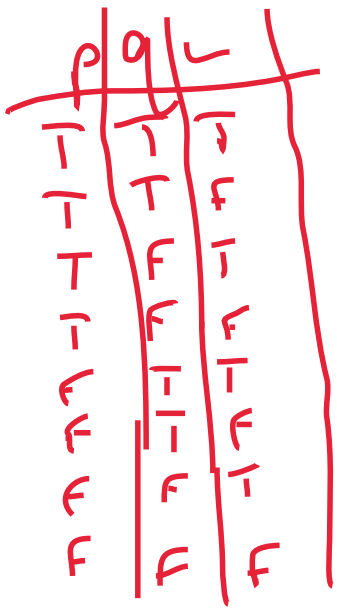
$\wedge \approx \times$

$\vee \approx +$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p(q+r) = pq + pr$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$$



Basic laws of logic

$$-(a+b) = (-a)+(-b)$$

- Double negation:
- Distributive laws:
- DeMorgan's laws:

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

Application: Negating an implication

$p \rightarrow q \equiv \neg p \vee q$

Ex. 1: If s is a square, then s is a rectangle.

(p: s is a square; q: s is a rectangle)

Negation:

$$\neg(p \rightarrow q)$$
$$\equiv \neg(\neg p \vee q)$$
$$\equiv \neg(\neg p) \wedge \neg q$$
$$\equiv p \wedge \neg q$$

← remember

Application: Logical equivalence

Are the following two statements logically equivalent?

- $r \rightarrow (p \wedge \neg q)$
- $(\neg r \vee p) \wedge (\neg r \vee \neg q)$

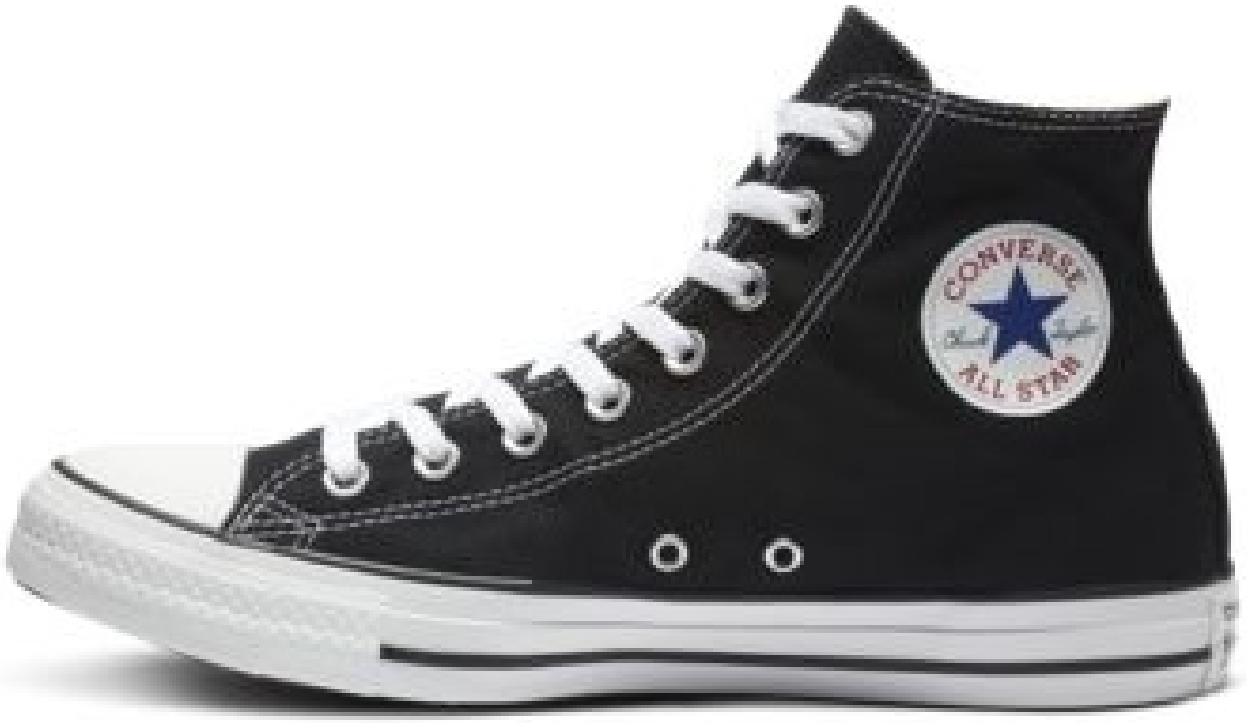
Yes!

$r \rightarrow (p \wedge \neg q) \equiv \neg r \vee (p \wedge \neg q)$

$\equiv (\neg r \vee p) \wedge (\neg r \vee \neg q)$

Converse

Converse



Converse

Reversal of an implication

Converse

Reversal of an implication

Ex. 1: If s is a square, then s is a rectangle.

Converse:

p

q

Original: $p \to q$

Converse: $q \to p$

$q \to p \neq p \to q$

$p \leftrightarrow q$

"bi-conditional"

If s is a rectangle, then s is a square.

Contrapositive

Reverse implication and negate both sides

Contrapositive

Reverse implication and negate both sides

Ex. 1: If s is a square, then s is a rectangle.

Contrapositive:

p q

$p \rightarrow q$

$\equiv$

$\neg q \rightarrow \neg p$

$\neg p \vee q$

$\neg(\neg q) \vee \neg p$

$q \vee \neg p$

$\neg p \vee q$

If s is not a rectangle,

then s is not a square.

Recap: Learning Objectives

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