

Introduction to Logic

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Learning Objectives

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By the end of this lesson, you will be able to:

- Define and distinguish propositional and predicate logic.
- Recall the definitions of logical operators, quantifiers, and related terminology.

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- “waffles” (sentence fragment)
- $9 + 10$ (just a mathematical expression; can't be true/false)

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Warning

In English, we often use the word “or” to mean *exclusive* or. In this class, assume “or” means *inclusive* or unless told otherwise.

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- More logical equivalences in Chapter 2

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- When variables are fixed, becomes a proposition
 - Combine with quantifiers to make more general statements

Quantifiers

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- 3 The *unique existence* quantifier: “there is a unique”
Example: There is a unique natural number y such that $y^2 + y = 12$.

Recall: Learning Objectives

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