Introduction to Logic

lan Ludden

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• Define and distinguish propositional and predicate logic.

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- Define and distinguish propositional and predicate logic.
- Recall the definitions of logical operators, quantifiers, and related terminology.

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What is propositional logic?

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Non-examples

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Non-examples

- "waffles" (sentence fragment)
- 9 + 10 (just a mathematical expression; can't be true/false)

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Warning

In English, we often use the word "or" to mean *exclusive* or. In this class, assume "or" means *inclusive* or unless told otherwise.

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• Enumeration of all possible inputs and the operator's output

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- Enumeration of all possible inputs and the operator's output
- Can be used to define operators and prove logical equivalence

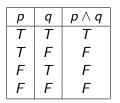
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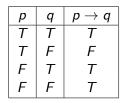
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p	q	$p \wedge q$
T	Т	Т
T	F	F
F	Т	F
F	F	F

p	q	p ightarrow q
T	Т	Т
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More logical equivalences in Chapter 2

What is predicate logic?

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Examples

•
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Examples

- $x^2 \ge 0$
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- When variables are fixed, becomes a proposition

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Examples

- $x^2 \ge 0$
- *y* is a six-legged animal
- *q* is prime
- When variables are fixed, becomes a proposition
- Combine with quantifiers to make more general statements

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Quantifiers

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Three quantifiers we care about

1 The *universal* quantifier: "for all/every" Example: For every integer x, $5x^2 \ge x$.

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- 1 The *universal* quantifier: "for all/every" Example: For every integer x, $5x^2 \ge x$.
- 2 The *existential* quantifier: "there exists" Example: There exists a rational number x such that $x^2 = 2$.

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- 2 The *existential* quantifier: "there exists" Example: There exists a rational number x such that $x^2 = 2$.
- **3** The *unique existence* quantifier: "there is a unique" Example: There is a unique natural number y such that $y^2 + y = 12$.

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