Introduction to Logic

Ian Ludden

Learning Objectives

By the end of this lesson, you will be able to:

Learning Objectives

By the end of this lesson, you will be able to:

- Define and distinguish propositional and predicate logic.

Learning Objectives

By the end of this lesson, you will be able to:

- Define and distinguish propositional and predicate logic.
- Recall the definitions of logical operators, quantifiers, and related terminology.

What is propositional logic?

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

Examples

- "My house has five windows." (False)

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

Examples

- “My house has five windows.” (False)
- “The principal square root of 25 is 5.” (True)

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

Examples

- "My house has five windows." (False)
- "The principal square root of 25 is 5." (True)
- $9 + 10 = 21$ (False)

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

Examples

- "My house has five windows." (False)
- "The principal square root of 25 is 5." (True)
- $9 + 10 = 21$ (False)

Non-examples

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

Examples

- “My house has five windows.” (False)
- “The principal square root of 25 is 5.” (True)
- $9 + 10 = 21$ (False)

Non-examples

- “waffles” (sentence fragment)

What is propositional logic?

Definition

A *proposition* is a true/false statement. The field of *propositional logic* is the study of propositions.

Examples

- "My house has five windows." (False)
- "The principal square root of 25 is 5." (True)
- $9 + 10 = 21$ (False)

Non-examples

- "waffles" (sentence fragment)
- $9 + 10$ (just a mathematical expression; can't be true/false)

Building complex propositions

Suppose p and q are propositions. Use logical operators:

- “not” ($\neg p$): true exactly when p is false

Building complex propositions

Suppose p and q are propositions. Use logical operators:

- "not" ($\neg p$): true exactly when p is false
- "inclusive or" ($p \vee q$): true when at least one of p and q is true

Building complex propositions

Suppose p and q are propositions. Use logical operators:

- "not" ($\neg p$): true exactly when p is false
- "inclusive or" ($p \vee q$): true when at least one of p and q is true
- "and" ($p \wedge q$): true when both p and q are true

Building complex propositions

Suppose p and q are propositions. Use logical operators:

- "not" ($\neg p$): true exactly when p is false
- "inclusive or" ($p \vee q$): true when at least one of p and q is true
- "and" ($p \wedge q$): true when both p and q are true
- "exclusive or" ($p \oplus q$): true when *exactly* one of p and q is true

Building complex propositions

Suppose p and q are propositions. Use logical operators:

- "not" ($\neg p$): true exactly when p is false
- "inclusive or" ($p \vee q$): true when at least one of p and q is true
- "and" ($p \wedge q$): true when both p and q are true
- "exclusive or" ($p \oplus q$): true when *exactly* one of p and q is true
- "implies" ($p \rightarrow q$): true if q is true or p is false (or both)

Building complex propositions

Suppose p and q are propositions. Use logical operators:

- "not" ($\neg p$): true exactly when p is false
- "inclusive or" ($p \vee q$): true when at least one of p and q is true
- "and" ($p \wedge q$): true when both p and q are true
- "exclusive or" ($p \oplus q$): true when *exactly* one of p and q is true
- "implies" ($p \rightarrow q$): true if q is true or p is false (or both)

Warning

In English, we often use the word "or" to mean *exclusive* or. In this class, assume "or" means *inclusive* or unless told otherwise.

Truth tables

- Enumeration of all possible inputs and the operator’s output

Truth tables

- Enumeration of all possible inputs and the operator's output
- Can be used to define operators and prove logical equivalence

Truth tables

- Enumeration of all possible inputs and the operator's output
- Can be used to define operators and prove logical equivalence

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

$p \to q \equiv \neg p \vee q$

Truth tables

- Enumeration of all possible inputs and the operator's output
- Can be used to define operators and prove logical equivalence

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

- More logical equivalences in Chapter 2

What is predicate logic?

What is predicate logic?

Definition

A *predicate* is a statement whose truth value is determined by assignments to its variables.

What is predicate logic?

Definition

A *predicate* is a statement whose truth value is determined by assignments to its variables.

Examples

- $x^2 \geq 0$

What is predicate logic?

Definition

A *predicate* is a statement whose truth value is determined by assignments to its variables.

Examples

- $x^2 \geq 0$
- y is a six-legged animal

What is predicate logic?

Definition

A *predicate* is a statement whose truth value is determined by assignments to its variables.

Examples

- $x^2 \geq 0$
- y is a six-legged animal
- q is prime

What is predicate logic?

Definition

A *predicate* is a statement whose truth value is determined by assignments to its variables.

Examples

- $x^2 \geq 0$
- y is a six-legged animal
- q is prime

- When variables are fixed, becomes a proposition

What is predicate logic?

Definition

A *predicate* is a statement whose truth value is determined by assignments to its variables.

Examples

- $x^2 \geq 0$
- y is a six-legged animal
- q is prime

- When variables are fixed, becomes a proposition
- Combine with quantifiers to make more general statements

Quantifiers

Quantifiers

Definition

A *quantifier* quantifies (gives a number of) how many elements in the universe make the claim true.

Definition

A *quantifier* quantifies (gives a number of) how many elements in the universe make the claim true.

Three quantifiers we care about

1. The *universal* quantifier: "for all/every"
   Example: For every integer x, $5x^2 \geq x$.

Quantifiers

Definition

A *quantifier* quantifies (gives a number of) how many elements in the universe make the claim true.

Three quantifiers we care about

1. The *universal* quantifier: "for all/every"
   Example: For every integer x, $5x^2 \geq x$.
2. The *existential* quantifier: "there exists"
   Example: There exists a rational number x such that $x^2 = 2$.

$x = \pm\sqrt{2}$

Quantifiers

Definition

A *quantifier* quantifies (gives a number of) how many elements in the universe make the claim true.

Three quantifiers we care about

1. The *universal* quantifier: “for all/every”
   Example: For every integer x, $5x^2 \geq x$.
2. The *existential* quantifier: “there exists”
   Example: There exists a rational number x such that $x^2 = 2$.
3. The *unique existence* quantifier: “there is a unique”
   Example: There is a unique natural number y such that $y^2 + y = 12$.

$y^2 + y - 12 = 0$

$(y+4)(y-3) = 0 \Rightarrow$ ~~$y = -4$~~ or $y = 3$

Recall: Learning Objectives

By the end of this lesson, you will be able to:

- Define and distinguish propositional and predicate logic.
- Recall the definitions of logical operators, quantifiers, and related terminology.