CS 173, Fall 2012
Midterm 2, 6 November 2012

NAME:

NETID (e.g. hpotter23, not 123987654):

DISCUSSION DAY:

DISCUSSION TIME:

You will lose a point if you don't accurately write the day and time of the discussion you are officially registered for. You may look this up in the rosters at the podium before turning in your exam.

We will be checking photo ID's during the exam. Have your ID handy.
(Forgot your ID? Don't panic: we have a copy of the photo roster.)

Turn in your exam at the front when you are done.

You have 75 minutes to finish the exam.

Problem	1	2	3	4	5	total
Possible	14	15	13	13	15	70
Score						

INSTRUCTIONS (read carefully)

- There are 5 problems, each on its own page. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and in the table on the cover page.
- Points may be deducted for solutions which are correct but hard to read, hard to understand, poorly explained, or excessively complicated.
- Brief explanations and/or showing work, even when not formally required, may increase partial credit for buggy answers.
- Except where explicitly indicated, it isn't necessary to simplify or calculate out complex constant expressions such as $(0.7)^3(0.3)^5$, $\frac{0.15}{3.75}$, 3^{17}, $7!$, and the like.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam. No notes or electronic devices of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem 1: Checkbox (14 points)

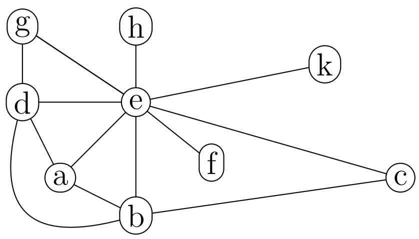
Check the box that best characterizes each item.

$\exists x \in \mathbb{Z},\ \forall y \in \mathbb{Z},\ x \geq y$	**true** ☐ **false** ☐
I found 143 marbles in my saucepan last Saturday. 143 is ______ the number of marbles that fits in my saucepan	**exactly** ☐ **a lower bound on** ☐ **an upper bound on** ☐
Number of nodes at level k in a full complete binary tree.	2^k ☐ $2^k - 1$ ☐ $2^{k+1} - 1$ ☐ 2^{k-1} ☐
$f : \mathbb{Z} \to \mathbb{R}$ $f(x) = 2x$ The codomain of f is	$\mathbb{Z}$ ☐ $\mathbb{R}$ ☐ **{even integers}** ☐ $2x$ ☐
The shortest possible cycles have	**0 nodes** ☐ **1 node** ☐ **2 nodes** ☐ **3 nodes** ☐
The diameter of the wheel graph W_5	**1** ☐ **2** ☐ **3** ☐ **4** ☐ **5** ☐ **6** ☐
The number of edges in K_n (complete graph on n nodes)	n ☐ $\frac{n(n-1)}{2}$ ☐ $\frac{n(n+1)}{2}$ ☐ $\frac{n}{2}$ ☐

Problem 2: Short answer (15 points)

(a) (4 points) How many different 11-letter strings can I make by permuting the letters in the 11-letter word "confessions"?

(b) (6 points) What is the chromatic number of graph G (below)? Justify your answer.



(c) (5 points) Using the same graph G as in part (b), how many isomorphisms are there from G to itself? Justify your answer.

Problem 3: Functions (13 points)

(a) (5 points) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{\text{red}, \text{green}, \text{blue}\}$. Give an example of a function $f : X \rightarrow B$ where $X \subseteq A$ and f is not onto. You must say exactly what's in the set X and use a diagram to show which input values map to which output values. Do not attempt to build a defining equation for f.

(b) (8 points) Suppose that $g : \mathbb{N} \rightarrow \mathbb{N}$ is one-to-one. Let's define the function $f : \mathbb{N}^2 \rightarrow \mathbb{N}^2$ by $f(x, y) = (x + y, g(x))$. Prove that f is one-to-one.

Problem 4: Tree Induction (13 points)

Suppose that grammar G has these rules: $S \rightarrow SS \mid Sb \mid ab \mid b$

Suppose that the only start symbol is S (i.e. the root must have label S). And that the only terminals are a and b (i.e. a and b are the only possible labels for leaves). Finally, let's use $A(T)$ as shorthand for the number of a's in tree T, and $B(T)$ for the number of b's in T.

Use strong induction to prove that $A(T) \leq B(T)$ for any tree T matching grammar G.

The induction variable is named ______ and it is the ____________________ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Write your netID, in case this page gets pulled off:

Problem 5: Induction (15 points)

Let function $f : \mathbb{Z}^+ \to \mathbb{Z}$ be defined by

$$f(1) = 0$$

$$f(2) = 12$$

$$f(n) = 4 \cdot f(n-1) - 3 \cdot f(n-2), \quad \text{for } n \geq 3$$

Use strong induction on n to prove that $f(n) = 2 \cdot 3^n - 6$ for any positive integer n.

Base case(s):

Inductive hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: