Week 7: The Diagonal Robot

A robot is walking around on the 2D integer grid. It starts at (1,1), and at each step it moves to one of the closest diagonal grid points - e.g. its first step can take it to any of (2,2), (2,0), (0,0), or (0,2). Prove that the robot can never reach the point (0,1).

Hint:

1. First, draw a picture to make sure the problem statement makes sense, and experiment with what the robot can reach in a few steps. For example, find a sequence of steps that allows the robot to reach (−3,3).

2. Based on patterns you see in step 1, guess some property which is true of all points the robot can reach - as a (wrong) example, you might guess that every (x,y) point the robot can reach will satisfy x ≤ y.

3. Prove by induction that your guess from the previous step is correct. Your induction variable should be the number of steps the robot takes.

4. Conclude by showing that (0,1) does not have your proven property, so it must not be reachable.