Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Prove the following claim, using your best mathematical style and working directly from the definition of "divides." Hint: look at remainders and use proof by cases. You may use the fact that if $a \mid b$, then $a \mid bc$ for any integers a, b, and c.

For any integer n, $n^4 - n^2$ is divisible by 3.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For all integers p and q, if $p^2(q^2 - 4)$ is odd, then p and q are odd.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:__________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Prove the following claim, using direct proof and your best mathematical style.

For any integers m and k, if $k \leq 7$ and $0 < m - 3 \leq \frac{k}{7}$, then $m^2 - 9 \leq k$.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) The following claim is slightly buggy. Explain briefly what values cause it to fail, add one extra (very simple) condition to the hypothesis. Then prove your revised claim, working directly from the definition of "divides" and using your best mathematical style.

For any integers a, b, and c, if $a^2b \mid cb$, then $a \mid c$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) For any two real numbers x and y, the arithmetic mean is $M(x,y) = \frac{x+y}{2}$ and the harmonic mean is $H(x,y) = \frac{2xy}{x+y}$. Use proof by contrapositive to prove the following claim, using these definitions and your best mathematical style.

For all real numbers x and y ($x \neq -y$), if $x \neq y$, then $H(x,y) \neq M(x,y)$.

You must begin by explicitly stating the contrapositive of the claim:

Name:____

NetID:_______________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) For any two real numbers x and y, the Harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Use this definition and your best mathematical style to prove the following claim:

For all positive real numbers x and y and p, if $x \geq y$, then $H(x, p) \geq H(y, p)$.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

For all integers a, b, c, p and k (c positive), if $ap \equiv b \pmod{c}$ and $k \mid a$ and $k \mid c$, then $k \mid b$.

Name:____

NetID:__________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Notice that, for any integer p, $\lfloor p \rfloor = \lfloor p + \frac{1}{2} \rfloor = p$. Using this fact and your best mathematical style, prove the following claim:

For any integer n, if n is odd, then $\left\lfloor \frac{n}{2} \right\rfloor^2 + \left\lfloor \frac{n}{2} \right\rfloor \geq \frac{1}{2} \left\lfloor \frac{n^2}{2} \right\rfloor$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a - b = nk$ for some integer n.

Claim: For all integers a, b, c, d, j and k (j and k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ and $j \mid k$, then $a + c \equiv b + d \pmod{j}$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6** **7**

(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers p and q ($p \neq -1$), if $\frac{2}{p+1}$ and $p + q$ are rational, then q is rational.