CS 173, Spring 19 Examlet 13

Name:____

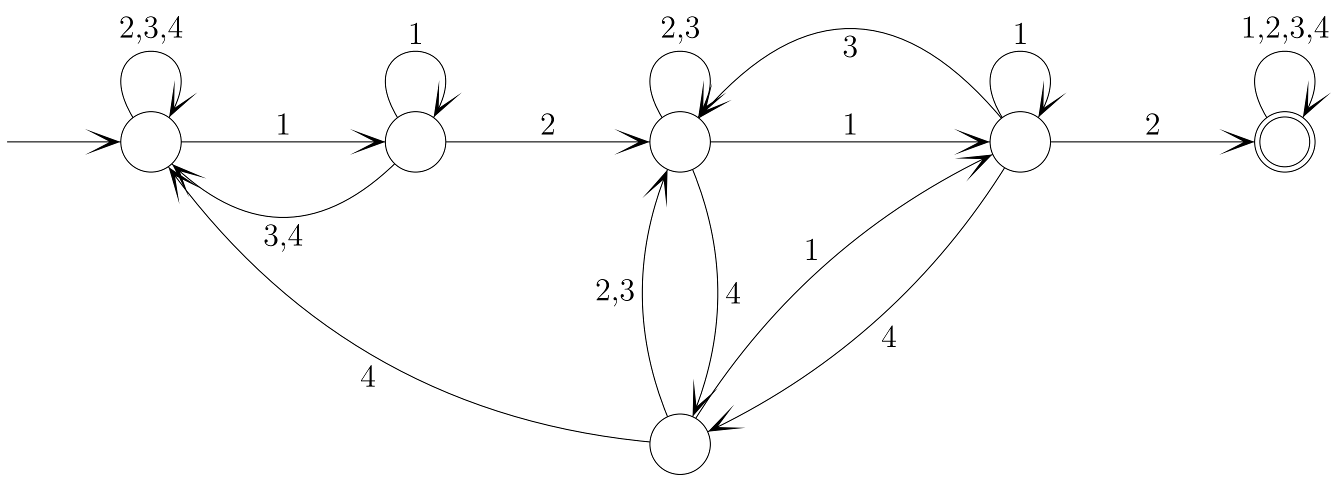
NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

(15 points) Q needs your help designing an exploding keychain. The keychain has four buttons, labelled 1, 2, 3, and 4. To make it explode, James Bond must input 12 twice. The two copies of 12 could be together (1212) or separated by other digits (1234312). Your state machine should move into an end state when that happens and remain in that final state as further digits come in. Exception: if you aren't already in the end state, two consecutive 4's (44) should abort the command (i.e. put the controller back in the start state). For efficiency, the state machine must be deterministic. Specifically, if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

Solution:



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(5 points) A DNA molecule can be modelled as a finite sequence of base pairs. Each base pair consists of two nucleobases. There are four possible nucleobases: A, T, G, and C. Is the set of all DNA molecules countable or uncountable?

Solution: It is countable. Each molecule is a finite sequence of base pairs, and there are only finitely many base pairs. So this is just like the set of finite-length strings made from a finite alphabet.

(10 points) Check the (single) box that best characterizes each item.

The set $\mathbb{Q}^2$	finite ☐	countably infinite ☑	uncountable ☐
$\lvert A \times A\rvert \geq \lvert A\rvert$	true ☑	false ☐	true for some sets ☐
The set of all finite lists of integers.	finite ☐	countably infinite ☑	uncountable ☐
$\mathbb{R} - \mathbb{Q}$	finite ☐	countably infinite ☐	uncountable ☑
Any function from $\mathbb{N}$ to $\{0, 1\}$ has a corresponding C++ program that computes it.	true ☐	false ☑	not known ☐

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(5 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

For any sets A, B, and C, if $A \cap B = \emptyset$ and $B \cap C = \emptyset$ then $A \cap C = \emptyset$.

Solution: This claim is false. Consider $A = C = \{1\}$ and $B = \{2\}$. Then $A \cap B = \emptyset$ and $B \cap C = \emptyset$, but $A \cap C = \{1\} \neq \emptyset$.

(10 points) Check the (single) box that best characterizes each item.

$p \rightarrow q \equiv \neg p \rightarrow \neg q$	true ☐	false ☑		
$2 \mid -4$	true ☑	false ☐		
Chromatic number of a graph containing a W_7.	≥ 3 ☐	≥ 4 ☑	≥ 7 ☐	can't tell ☐
$g : \mathbb{Z} \rightarrow \mathbb{R}$ $g(x) = x + 2.137$	one-to-one ☑	not one-to-one ☐	not a function ☐	
Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. Each slogan appears on at least two shirts.	true ☐	false ☑		

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(5 points) Suppose that A and B are disjoint sets, C_A is a partition of A and C_B is a partition of B. Is $C_A \cup C_B$ a partition of $A \cup B$? Briefly justify your answer.

Solution: Yes. Each element of A belongs to exactly one element of C_A. Each element of B belongs to exactly one element of C_B. Since A and B are disjoint, there can't be any partial overlap between the elements of C_A and C_B. Since neither C_A nor C_B contains the empty set, their union can't contain it either.

(10 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $O(g(n))$. Will $f(n)$ be $\Theta(g(n))$?	no ☐	sometimes ☑	yes ☐

Total number of leaves in a full and complete 5-ary tree of height h	5^h ☑	$\leq 5^h$ ☐	$\geq 5^h$ ☐	$5^{h+1} - 1$ ☐

The diameter of the 4-dimensional hypercube Q_4	1 ☐	2 ☐	4 ☑	16 ☐

$T(1) = d$	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐
$T(n) = 3T(n/2) + d$	$\Theta(n^{\log_3 2})$ ☐	$\Theta(n^{\log_2 3})$ ☑	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$\mid \mathbb{P}(\{4,5,6,7,8\} \times \emptyset) \mid$	$\emptyset$ ☐	$\{\emptyset\}$ ☐	0 ☐	1 ☑	25 ☐	2^5 ☐