(10 points) Suppose we have a function $f$ defined (for $n$ a power of 4) by

$$
\begin{align*}
    f(1) &= 0 \\
    f(n) &= 2f(n/4) + n \text{ for } n \geq 4
\end{align*}
$$

Your partner has already figured out that

$$
    f(n) = 2^k f(n/4^k) + n \sum_{p=0}^{k-1} 1/2^p
$$

Finish finding the closed form for $f(n)$ assuming that $n$ is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$. 

(10 points) Suppose we have a function $F$ defined (for $n$ a power of 2) by

$$F(2) = c$$

$$F(n) = F(n/2) + n \text{ for } n \geq 4$$

Your partner has already figured out that

$$F(n) = F(n/2^k) + \sum_{i=0}^{k-1} n \frac{1}{2^i}$$

Finish finding the closed form for $F$. Show your work and simplify your answer.
1. (8 points) Suppose we have a function $f$ defined by

$$\begin{align*}
f(0) &= f(1) = 3 \\
f(n) &= 5f(n-2) + d, \text{ for } n \geq 2
\end{align*}$$

where $d$ is a constant. Express $f(n)$ in terms of $f(n-6)$ (where $n \geq 6$). Show your work and simplify your answer. You do not need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The chromatic number of the 4-dimensional hypercube $Q_4$  

2 3 4 5
(10 points) Suppose we have a function $f$ defined (for $n$ a power of 4) by

$$
\begin{align*}
f(1) &= 0 \\
f(n) &= 2f(n/4) + n \text{ for } n \geq 4
\end{align*}
$$

Express $f(n)$ in terms of $f(n/4^{13})$ (assuming $n$ is large enough that this input hasn’t reached the base case). Express your answer using a summation and show your work. Do not finish the process of finding the closed form for $f(n)$. 

1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 3) by

\[
\begin{align*}
g(1) &= c \\
g(n) &= 3g(n/3) + n \text{ for } n \geq 3
\end{align*}
\]

Express $g(n)$ in terms of $g(n/3^3)$ (where $n \geq 27$). Show your work and simplify your answer. You do not need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The number of nodes in the 4-dimensional hypercube $Q_4$ 4 [ ] 16 [ ] 32 [ ] 64 [ ]
(10 points) Suppose we have a function $g$ defined (for $n$ a power of 4) by

$$
\begin{align*}
g(1) &= c \\
g(n) &= 2g(n/4) + n \text{ for } n \geq 4
\end{align*}
$$

Your partner has already figured out that

$$
g(n) = 2^k g(n/4^k) + n \sum_{p=0}^{k-1} \frac{1}{2^p}
$$

Finish finding the closed form for $g(n)$ assuming that $n$ is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.
1. (8 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

\[
\begin{align*}
g(1) &= 1 \\
g(n) &= 4g(n/2) + n^2 \text{ for } n \geq 2
\end{align*}
\]

Your partner has already figured out that

\[
g(n) = 4^k g(n/2^k) + kn^2
\]

Finish finding the closed form for $g$. Show your work and simplify your answer.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n) = F(n - 1) + F(n - 2)$ for all integers $n \geq 0 \quad n \geq 1 \quad n \geq 2$
(10 points) Suppose we have a function $g$ defined (for $n$ a power of 2) by

\[
\begin{align*}
g(1) &= 3 \\
g(n) &= 4g(n/2) + n \text{ for } n \geq 2
\end{align*}
\]

Your partner has already figured out that

\[g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n2^p\]

Finish finding the closed form for $g(n)$ assuming that $n$ is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$. 