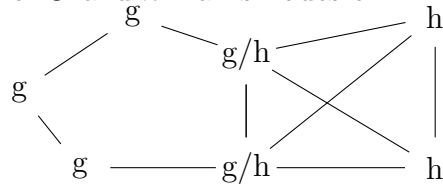


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1. (11 points) If  $G$  is a graph, recall that  $\chi(G)$  is its chromatic number. Suppose that  $G$  is a graph with at least one edge and  $H$  is another graph with at least one edge, not connected to  $G$ . Now, pick a specific edge  $e$  from  $G$  and an edge  $f$  from  $H$  and merge the two edges, creating a combined graph  $T$ . For example, suppose that  $G$  is  $C_5$  and  $H$  is  $K_4$ . Then  $T$  might look as follows, where  $g$  marks nodes of  $G$  and  $h$  marks nodes of  $H$ .



Describe how  $\chi(T)$  is related to  $\chi(G)$  and  $\chi(H)$ , justifying your answer. Your answer should handle any choice for  $G$  and  $H$ .

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{i=1}^{p-1} i$      $\frac{p(p-1)}{2}$          $\frac{(p-1)^2}{2}$          $\frac{p(p+1)}{2}$          $\frac{(p-1)(p+1)}{2}$    

Leal team's bridge held 100 pounds without collapsing. 100 pounds is \_\_\_\_\_ on how much the bridge can hold.

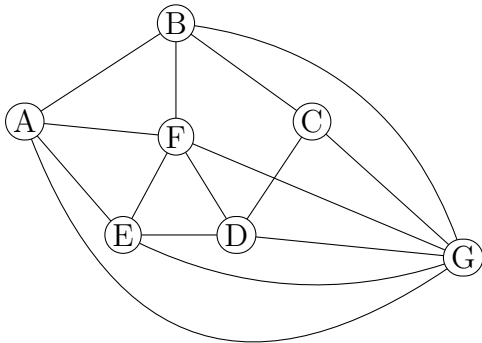
an upper bound on     exactly   
a lower bound on     not a bound on

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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $C_7$ .     $\geq 2$       $\geq 3$       $\leq 3$      can't tell

$\sum_{i=1}^{p-1} i$      $\frac{(p-1)^2}{2}$       $\frac{(p-1)(p+1)}{2}$       $\frac{p(p+1)}{2}$       $\frac{p(p-1)}{2}$

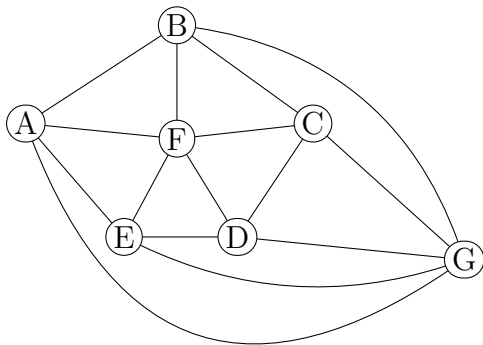
$\tau \leq 1.3$     an upper bound on  $\tau$      exactly  $\tau$    
    a lower bound on  $\tau$      not a bound on  $\tau$

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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $W_n$ .     $\leq 3$       $\geq 3$       $\geq n$      can't tell

$\sum_{i=1}^{p-1} \frac{i}{p}$      $\frac{p(p-1)}{2}$       $\frac{p(p+1)}{2}$       $\frac{(p+1)}{2}$       $\frac{(p-1)}{2}$

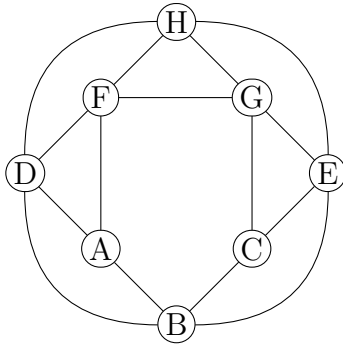
Putting 10 people in the canoe caused it to sink. 10 is \_\_\_\_\_ how many people the canoe can carry.    an upper bound on     exactly   
 a lower bound on     not a bound on

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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of an  
connected acyclic graph with 5 nodes.  $\leq 2$    $= 2$    $\leq 5$   can't tell

$\sum_{i=0}^{k-1} (k \cdot i + 2)$   $\frac{k^2(k+1)}{2} + 2k$    $\frac{k(k+1)}{2} + 2(k-1)$

$\frac{k^2(k-1)}{2} + 2k$    $\frac{k(k-1)}{2} + 2(k-1)$

$\pi \geq 1.3$  an upper bound on  $\pi$   exactly  $\pi$

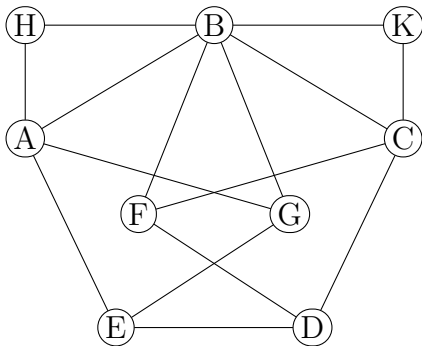
a lower bound on  $\pi$   not a bound on  $\pi$

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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a connected graph with 10 nodes.  $\leq 2$       $= 2$       $\geq 2$      can't tell

$\sum_{k=-2}^n k^2$       $\sum_{p=0}^{n+2} (p+2)^2$       $\sum_{p=0}^{n-2} (p-2)^2$       $\sum_{p=0}^{n+2} (p-2)^2$       $\sum_{p=0}^{n+2} p^2$

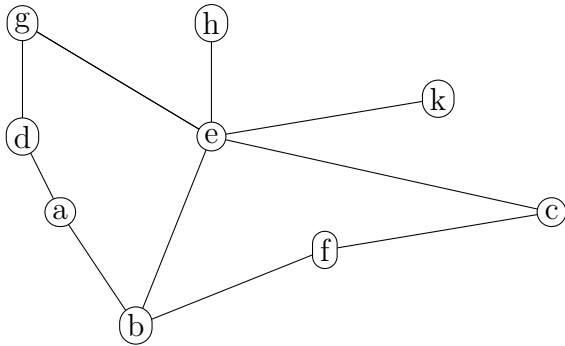
We have 30 tablespoons of filling. Each bun requires exactly one tablespoon of filling. 30 is \_\_\_\_\_ on how many buns we can make.    an upper bound on     exactly   
 a lower bound on     not a bound on

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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $W_7$ .     $\geq 3$       $\geq 4$       $\geq 7$      can't tell

$\sum_{k=1}^n k!$      $\sum_{p=0}^{n+1} (p+1)!$       $\sum_{k=0}^{n+1} (k-1)!$       $\sum_{k=0}^{n-1} (k+1)!$       $\sum_{p=0}^{n+1} k!$

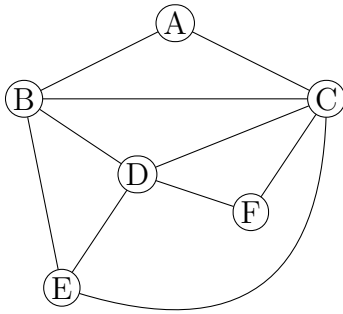
10 people rowed across Lake Tahoe in my canoe. 10 is \_\_\_\_\_ how many people the canoe can carry.    an upper bound on     exactly   
 a lower bound on     not a bound on

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1. (9 points) What is the chromatic number of the graph below? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph containing a  $K_n$ .  $\leq n$    $= n$    $\geq n$   can't tell

$\sum_{k=0}^n k!$   $\sum_{p=1}^{n+1} (p+1)!$    $\sum_{k=1}^{n+1} (k-1)!$    $\sum_{k=1}^{n-1} (k+1)!$    $\sum_{p=1}^{n+1} k!$

I heated 2 liters of milk in my big pot. 2 liters is \_\_\_\_\_ how much the pot holds. an upper bound on  exactly   
a lower bound on  not a bound on

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1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = x^2 - 4x + 3\}$$

$$B = \{(t + 2, t^2 - 1) : t \in \mathbb{R}\}$$

Prove that  $A = B$  by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of  $K_{m,n}$ .  
 (Assume  $m \geq 1, n \geq 1$ .)    2     3     4     can't tell

$\pi \leq 10$     an upper bound on  $\pi$      exactly  $\pi$    
 a lower bound on  $\pi$      not a bound on  $\pi$