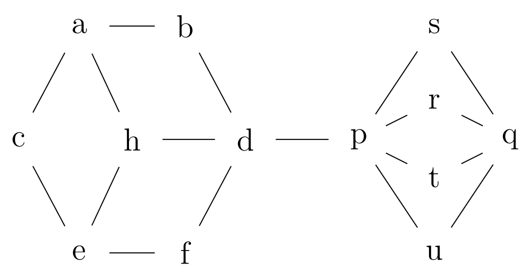
Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes d and p need to map to themselves. On the lefthand side, you can swap b and f (or not) and then the rest of the map is determined. On the righthand side, you can permute the nodes r, s, t, and u. So there are a total of $2 \cdot 4! = 48$ isomorphisms.

2. (5 points) The wheel graph W_{73} has 73 nodes on the rim. How many edges does it have?

Solution: It has 146 edges.

Name:____

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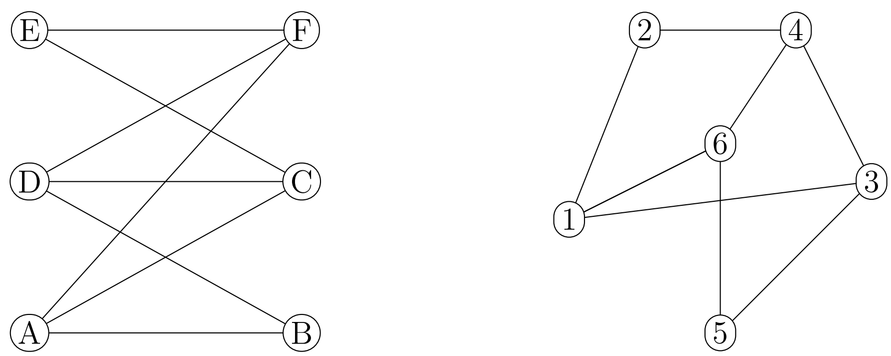
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X Graph Y

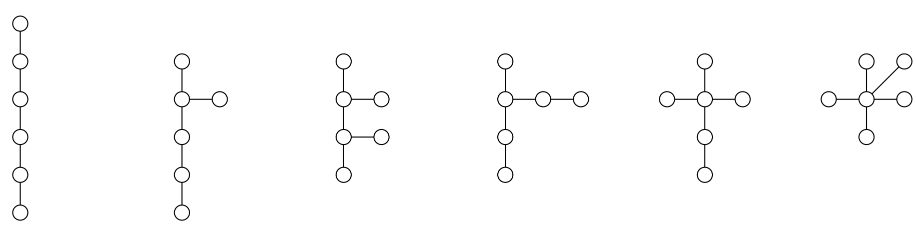


Solution: Yes, X and Y are isomorphic. We can map the nodes as follows

$f(A) = 6,\ f(B) = 5,\ f(C) = 1,\ f(D) = 3,\ f(E) = 2,\ f(F) = 4$

2. (5 points) Show four distinct (i.e. not isomorphic) graphs, each of which is connected and has six nodes and no cycles.

Solution: Any four out of the following set will work:



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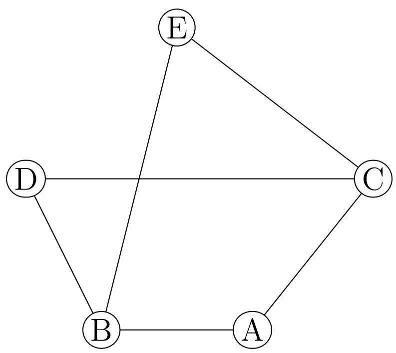
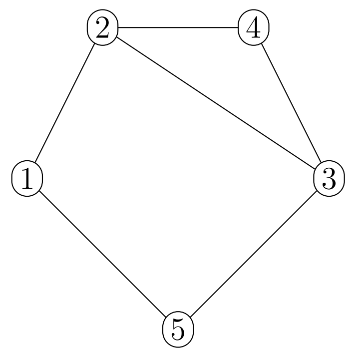
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



Graph Y



Solution: No, they are not isomorphic. Graph Y has a 5-cycle. Graph X is bipartite, so all of its cycles have even length.

2. (5 points) Is the cycle graph C_4 a subgraph of graph $K_{3,3}$? Briefly justify your answer.

Solution: Yes, it is. Pick two nodes on each side of $K_{3,3}$ and follow a path back-and-forth between the two sides.

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Name:____

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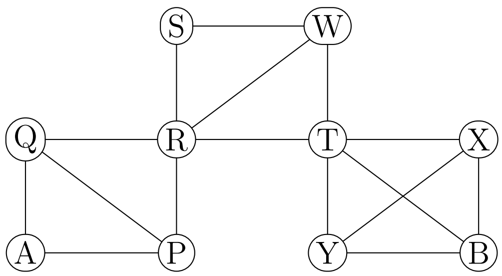
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes R and T must map to themselves. They are the only degree-5 nodes and only R is adjacent to a degree-2 node. So S, W, and A must also map to themselves. Q and P can be swapped. X, Y, and B can be permuted, creating 3! choices. So there are $2 \cdot 3!$ different isomorphisms of the whole graph.

2. (5 points) What is the difference between a path and an open walk?

Solution: A path uses each node only once. Open walks don't have to obey this constraint.

Name:____

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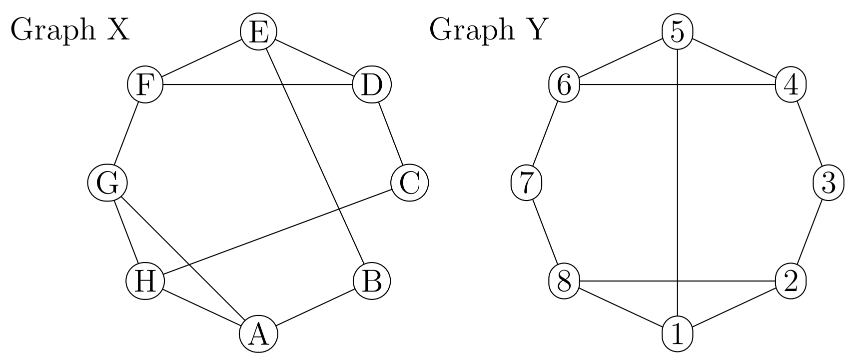
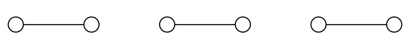


Solution: Yes, these graphs are isomorphic. Notice that the pairs of degree-2 nodes (B and C, 3 and 7) must be matched in one order or the other. Also notice that edge FG is distinctive because it connects the two triangles, so it must match 15 (in one direction or the other).

One possible map is f(C) = 7, f(B) = 3 for the degree-2 nodes. Then f(D) = 6, f(F) = 5, f(E) = 4 for one triangle. And f(H) = 8, f(A) = 2, and f(G) = 1 for the other triangle.

2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Suppose that graph G has degree sequence 1, 1, 1, 1, 1, 1. How many connected components does G have?

Solution: G must look as in the following picture. So G has three connected components.



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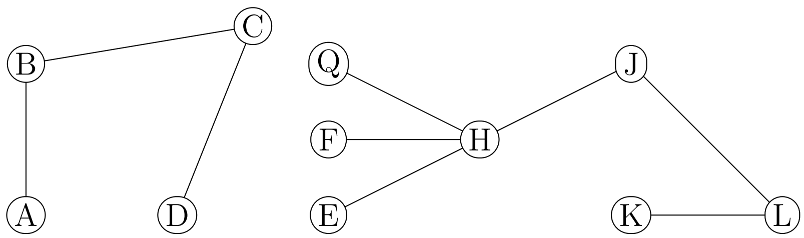
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: B and C can be swapped or not (two choices). This determines the matches for A and D.

The three degree-one nodes directly connected to H (E, F, and Q) can be rotated, giving us 3! more choices.

J, K, and L must match themselves.

So the total number of choices is $2 \cdot 3! = 12$.

2. (5 points) Is the cycle graph C_{17} a subgraph of the wheel graph W_{23}? Briefly justify your answer.

Solution: Yes, it is. Match 16 of the nodes in C_{17} with consecutive nodes on the rim of W_{23}. Then match the last node of C_{17} with the hub node of W_{23}.

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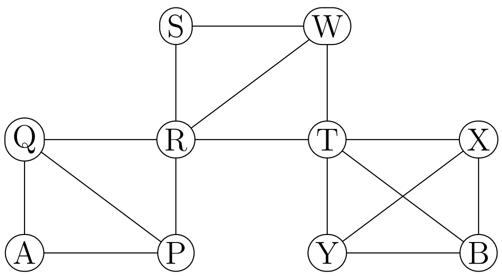
1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .



Solution: Nodes R and T must map to themselves. They are the only degree-5 nodes and only R is adjacent to a degree-2 node. So S, W, and A must also map to themselves. Q and P can be swapped. X, Y, and B can be permuted, creating 3! choices. So there are $2 \cdot 3!$ different isomorphisms of the whole graph.

2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Suppose graph G has degree sequence 1, 1, 1, 1, 2. Is G connected? Briefly justify your answer.

Solution: G isn't connected. The sum of the degrees is 6. So, by the Handshaking theorem, G has three edges. Suppose that we start with one base node. Connecting each of the other four nodes will require an edge, so at least four edges.

Or, if you prefer, G has to look as in the following picture. Two of the degree-1 nodes can be connected to the degree-2 node. But the only way to add two additional degree-1 nodes is to connect them to one another.

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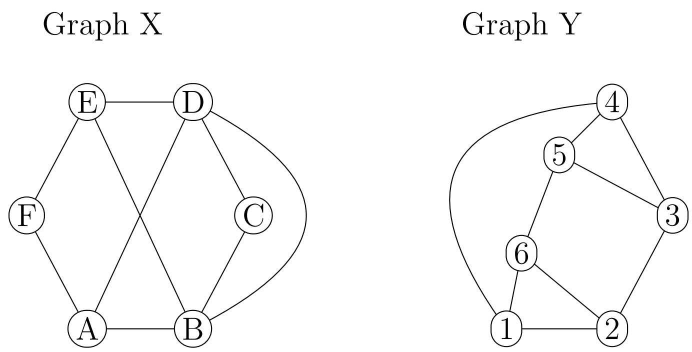
1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.



Solution: No, they are not isomorphic. Graph X has two nodes with degree 4. All the nodes in Graph Y have degree 3.

2. (5 points) Is the graph C_{10} bipartite? Briefly justify your answer.

Solution: Yes, it is bipartite. As you walk around the cycle, assign nodes to the two subsets in an alternating manner.