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1. (5 points) How many different 12-letter strings can be made by rearranging the letters in the word ‘‘apalachicola’’? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If  $f : \mathbb{Z} \rightarrow \mathbb{R}$  is a function such that  $f(x) = 2x$  then the set of all even integers is the \_\_\_\_\_ of  $f$ .

domain   
 image     co-domain   
 none of these

$f : \mathbb{N}^2 \rightarrow \mathbb{N}$   
 $f(p, q) = pq$

onto     not onto     not a function

$g : (\mathbb{Z}^+)^2 \rightarrow \mathbb{Z}^+$   
 $g(x, y) = \gcd(x, y)$

one-to-one     not one-to-one     not a function

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.

true     false

$\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}, y \leq x$

true     false

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1. (5 points) Hermione Grainger has 7000 socks in her magically expanding drawer. The socks are colored purple, magenta, and shocking pink. How many socks must she pull out of the drawer before she is guaranteed to have two socks of the same color. Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain.    true     false

$g : \mathbb{R} \rightarrow [-1, 1]$   
 $g(x) = \sin(x)$     onto     not onto     not a function

$g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$   
 $g(x, y) = (y, 3x)$     one-to-one     not one-to-one     not a function

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm.    true     false

$\exists t \in \mathbb{N}, \forall p \in \mathbb{Z}^+, \gcd(p, t) = p$     true     false

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1. (5 points) 8 presidential candidates (including Bernie and Hilary) need to line up for a photo. The new editor would like Bernie and Hilary to stand next to each other. How many different ways can we arrange the eight people?

2. (10 points) Check the (single) box that best characterizes each item.

Suppose  $f : A \rightarrow B$ . For all  $x, y \in A$ , if  $x = y$ , then  $f(x) = f(y)$ .    onto     one-to-one     neither

$g : \mathbb{R} \rightarrow [0, 1]$   
 $g(x) = \sin(x)$     onto     not onto     not a function

$f : \mathbb{R} \rightarrow \mathbb{Z}$   
 $f(x) = x$     one-to-one     not one-to-one     not a function

Each dorm room is given an integer access code between 1 and 10 (inclusive). According to the pigeon-hole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms.    true     false

$\exists y \in \mathbb{Z}, \forall x \in \mathbb{Z}, x - y < 100$     true     false

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1. (5 points) To make exam grading anonymous and therefore hopefully more fair, each of the 200 students in CS 241 has been assigned a unique 3-character exam code. The character set is  $\{\alpha, \beta, \gamma, \delta\}$ . Use the Pigeonhole Principle to explain what's wrong with this plan.

2. (10 points) Check the (single) box that best characterizes each item.

If  $f : \mathbb{Z} \rightarrow \mathbb{R}$  is a function such that  $f(x) = 2x$  then the real numbers is the \_\_\_\_\_ of  $f$ .  
 domain     co-domain   
 image     none of these

$f : \mathbb{Z} \rightarrow \mathbb{Z}$   
 $f(x) = x + 3$  ( $x$  even),  
 $f(x) = x - 22$  ( $x$  odd)  
 onto     not onto     not a function

$g : \mathbb{N}^2 \rightarrow \mathbb{N}$   
 $g(x, y) = \gcd(x, y)$   
 one-to-one     not one-to-one     not a function

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.  
 true     false

$\exists y \in \mathbb{R}^+, \forall x \in \mathbb{R}^+, xy = 1$   
 ( $\mathbb{R}^+$  is the positive real numbers.)  
 true     false

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1. (5 points) Suppose that  $|A| = p$  and  $|B| = q$ . How many different functions are there from  $A$  to  $B$ ?

2. (10 points) Check the (single) box that best characterizes each item.

A function is one-to-one if and only if each value in the co-domain has at most one pre-image.    true     false

$g : \mathbb{R}^2 \rightarrow \mathbb{R}$   
 $g(x, y) = \lfloor x \rfloor + y$     onto     not onto     not a function

$g : \mathbb{Z}^2 \rightarrow \mathbb{Z}^2$   
 $g(x, y) = (y, 3x)$     one-to-one     not one-to-one     not a function

Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. At least three of these shirts must have the same slogan.    true     false

$\forall x \in \mathbb{Q}, \exists m, n \in \mathbb{Z}, x = \frac{m}{n}$     true     false

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1. (5 points) 15 men and 15 women showed up to this week’s meeting of the UIUC Swing Dance Society. How many different ways can we form all of them into pairs, each pair containing one man and one woman?

2. (10 points) Check the (single) box that best characterizes each item.

Suppose  $f : A \rightarrow B$ . For all  $x \in A$ , there is a  $y \in B$ ,  $f(x) = y$ .    onto     one-to-one     neither

$g : (\mathbb{Z}^+)^2 \rightarrow \mathbb{Z}^+$   
 $g(x, y) = \gcd(x, y)$     onto     not onto     not a function

$f : \mathbb{N} \rightarrow \mathbb{R}$   
 $f(x) = x^2 + 2$     one-to-one     not one-to-one     not a function

Each ACM shirt has one of 6 trendy slogans. I bought 13 ACM shirts. There is a slogan that appears on at least two shirts.    true     false

$\forall x \in \mathbb{Z}, \exists y \in \mathbb{N}, x^2 = y$     true     false

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1. (5 points) Suppose that  $|A| = 2$  and  $|B| = 3$ . How many onto functions are there from  $A$  to  $B$ ? Briefly justify or show work.

2. (10 points) Check the (single) box that best characterizes each item.

If  $f : \mathbb{Z} \rightarrow \mathbb{R}$  is a function such that  $f(x) = 2x$  then the integers is the \_\_\_\_\_ of  $f$ .  
 domain     co-domain   
 image     none of these

$f : \mathbb{N}^2 \rightarrow \mathbb{Z}$   
 $f(p, q) = 2^p 3^q$     onto     not onto     not a function

$g : \mathbb{Z} \rightarrow \mathbb{Z}$   
 $g(x) = x|x|$     one-to-one     not one-to-one     not a function

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least one elf has stamina.    true     false

$\forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+, xy = 1$   
 ( $\mathbb{R}^+$  is the positive real numbers.)    true     false

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1. (5 points) Prof. Snape is teaching potions to 52 girls and 73 boys. Quiz 1 has integer scores between zero and 100 (inclusive). Assuming no one missed the quiz, what is the probability that two students got the same score? Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

If  $f : A \rightarrow B$  is one-to-one, then  $|A| \geq |B|$       $|A| \leq |B|$       $|A| = |B|$

$g : \mathbb{N}^2 \rightarrow \mathbb{N}$   
 $g(x, y) = \gcd(x, y)$     onto     not onto     not a function

$g : \mathbb{R} \rightarrow \mathbb{R}^2$   
 $g(x) = (x, 3x^2 + 2)$     one-to-one     not one-to-one     not a function

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes.    true     false

$\exists m, n \in \mathbb{Z}, \forall x \in \mathbb{Q}, x = \frac{m}{n}$     true     false