1. (10 points) Suppose that \( f : \mathbb{Z} \to \mathbb{Z} \) is onto. Let’s define \( g : \mathbb{Z}^2 \to \mathbb{Z} \) by \( g(x, y) = f(x - 7)f(y) \). Prove that \( g \) is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function \( g : C \to M \) to be “onto.” You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.
1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let’s define $g : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $g(x, y) = (2f(x) + f(y), f(x) - f(y))$. Prove that $g$ is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : C \to M$ to be “one-to-one.” You must use explicit quantifiers; do not use words like “unique.”
1. (10 points) Suppose that $A$ and $B$ are sets. Suppose that $f : B \to A$ and $g : A \to B$ are functions such that $f(g(x)) = x$ for every $x \in A$. Prove that $f$ is onto.

2. (5 points) Give an example of a function $f : \mathbb{N} \to \mathbb{N}$ which is one-to-one but not onto. Your answer must include a specific formula.
1. (10 points) Suppose that \( f : \mathbb{Z} \to \mathbb{Z} \) is onto. Let’s define \( g : \mathbb{Z}^2 \to \mathbb{Z} \) by \( g(x, y) = f(x) + 2f(y) - 6 \). Prove that \( g \) is onto.

2. (5 points) \( A = \{0, 2, 4, 6, 8, 10, 12, \ldots\} \), i.e. the even integers starting with 0.
   \( B = \{1, 4, 9, 16, 25, 36, 49, \ldots\} \), i.e. perfect squares starting with 1.
   Give a specific formula for a bijection \( f : A \to B \). (You do not need to prove that it is a bijection.)
1. (10 points) Suppose that $f : A \to B$ and $g : B \to C$ are one-to-one. Prove that $g \circ f$ is one-to-one.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : \mathbb{R} \to \mathbb{R}$ to be "increasing." You must use explicit quantifiers.
1. (10 points) Let $P$ be the set of pairs of positive integers. Suppose that $f : P \rightarrow \mathbb{R}^2$ is defined by $f(x, y) = \left(\frac{x}{y}, x + y\right)$. Prove that $f$ is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{N} \rightarrow \mathbb{N}$ which is onto but not one-to-one. Your answer must include a specific formula.
1. (10 points) Suppose that $f : \mathbb{Z}^2 \to \mathbb{Z}$ is defined by $f(x, y) = 3x + 5y$. Prove that $f$ is onto.

2. (5 points) $A = \{0, 1, 4, 9, 16, 25, 36, \ldots\}$, i.e. perfect squares starting with 0.
   
   $B = \{2, 4, 6, 8, 10, 12, 14, \ldots\}$, i.e. the even integers starting with 2.

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)
1. (10 points) Let’s define the function \( f : \mathbb{R}^+ \rightarrow \mathbb{R} \) by \( f(x) = \frac{4x - 1}{2x + 5} \). (\( \mathbb{R}^+ \) is the positive reals.) Prove that \( f \) is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function \( g : \mathbb{R} \rightarrow \mathbb{R} \) to be “strictly increasing.” You must use explicit quantifiers.