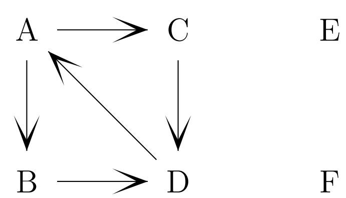
CS 173, Spring 19 Examlet 4, white

Name:____

NetID:______________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑ (√)

Symmetric: ☐ Antisymmetric: ☑ (√)

Transitive: ☐

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that $\sim$ is the relation on S where $a \sim b$ if and only if a and b contain the same number of 1's. For example, `0101` $\sim$ `1000001`. List three members of [`111`].

   Solution: For example, `111`, `1101`, and `01110`.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ or $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution:

   This relation is not antisymmetric. We have $(0, 1)T(1, 0)$ and $(1, 0)T(0, 1)$, but $(0, 1) \neq (1, 0)$.

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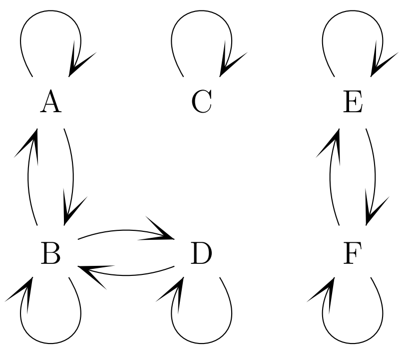

Reflexive: [√] Irreflexive: [ ]

Symmetric: [√] Antisymmetric: [ ]

Transitive: [ ]

2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, antisymmetric, transitive

3. (5 points) Suppose that R is a relation on the integers such that xRy for all integers x and y. Is R an equivalence relation?

   Solution: Yes, R is an equivalence relation. All three properties of an equivalence relation (reflexive, symmetric, transitive) are requiring that certain pairs be related (e.g. unlike antisymmetry which requires that certain pairs not be related). So they have to be true because all pairs are related.

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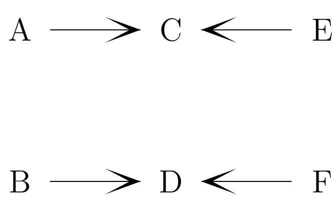
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	√
Symmetric:	☐	Antisymmetric:	√
Transitive:	√		

2. (5 points) Let's define the ~~equivalence~~ relation $\sim$ on $\mathbb{N}^3$ such that $(x, y, z) \sim (p, q, r)$ if and only $(x, y, z) = \alpha(p, q, r)$ for some integer α. ~~List three members of [(1, 2, 3)]~~ List three elements that are related to $(1, 2, 3)$ in either direction.

   Solution: For example $(1, 2, 3)$, $(-1, -2, -3)$, $(6, 12, 18)$.

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $|a - b| \leq 13$ Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, it is not transitive. Consider $a = 0$, $b = 13$, $c = 26$. Then aRb and bRc. 'However, $|a - c| = 26$, so $a \not R c$.

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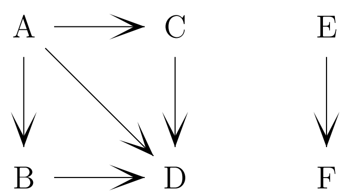
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: [ ] Irreflexive: [√]

Symmetric: [ ] Antisymmetric: [√]

Transitive: [√]

2. (5 points) Let R be the relation on the integers such that xRy if and only if $\lfloor x/4 \rfloor = \lfloor y/4 \rfloor$. List the values in [8].

   Solution: [8] contains (only) 8, 9, 10, and 11

3. (5 points) Let T be a reflexive relation defined on the integers. Let S be the relation on the integers such that aSb if and only if there is an integer k such that aTk and kTb. Is S reflexive? (I.e. is S reflexive for any reflexive relation T?) Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: S is always reflexive. Pick any integer x. xTx because T is reflexive. If we let $k = x$, then we also have xTk and kTx. So xSx.

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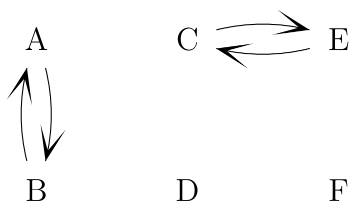
Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☑ Antisymmetric: ☐

Transitive: ☐

2. (5 points) Can a relation be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. However, this must be the empty relation in which no elements are related. (No edges in the graph representation.) If we have aRb, then we must have bRa (by symmetry) and so aRa (by transitivity), which is inconsistent with the relation being irreflexive.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $x = y$. Is R a partial order?

   Solution: Yes, R is a partial order. It's reflexive because elements are always related to themselves. The only way to match the hypothsis of transitive is for all three integers to be equal, which makes the conclusion true. So it's transitive. It is anti-symmetric by vacuous truth.

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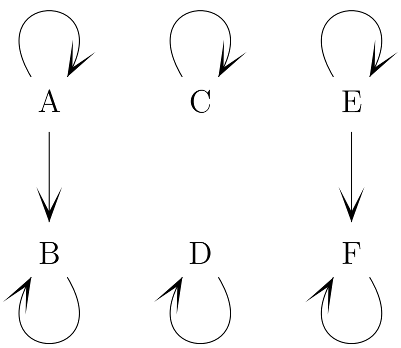
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	$\sqrt{}$	Irreflexive:	
Symmetric:		Antisymmetric:	$\sqrt{}$
Transitive:	$\sqrt{}$		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be symmetric.

   Solution: For any $x, y \in A$, if xRy, then yRx.

3. (5 points) Let S be the relation defined on $\mathbb{N}$ such that aTb if and only if $a = b + 2k$ for some natural number k. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is antisymmetric. Notice that k cannot be negative, so if aTb then $a \leq b$. So two numbers can be related in both directions only if they are equal.

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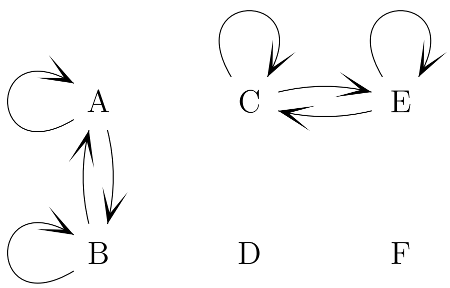
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☑ $\surd$ Antisymmetric: ☐

Transitive: ☑ $\surd$

2. (5 points) Let's define the equivalence relation $\sim$ on $\mathbb{N}^3$ such that $(x, y, z) \sim (p, q, r)$ if and only $x + y + z = p + q + r$. List three members of $[(1, 2, 3)]$.

   Solution: The three coordinates need to be non-negative integers that sum to 6. So some example members are $(1, 2, 3)$, $(3, 2, 1)$, and $(3, 0, 3)$.

3. (5 points) Suppose that R is a relation on the integers such xRy if and only if $xy = 1$ for all integers x and y. Is R a partial order?

   Solution: No, R is not a partial order. Notice that the only relations are when $x = y = 1$ or $x = y = -1$. So it's transitive and antisymmetric, but not reflexive.

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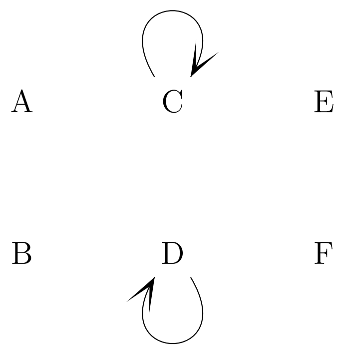

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☑ $\surd$ Antisymmetric: ☑ $\surd$

Transitive: ☑ $\surd$

2. (5 points) A relation is an equivalence relation if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, symmetric, transitive

3. (5 points) Suppose that $\succeq$ is the relation between subsets of the integers such that $A \succeq B$ if and only if $A - B \neq \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is $\succeq$ transitive? Informally explain why it's true or give a concrete counter-example.

   Solution: $\succeq$ is not transitive. Consider $A = C = \{3\}$ and $B = \{4\}$. Then $A - B = \{3\}$ and $B - C = \{4\}$. So $A \succeq B$ and $B \succeq C$. But $A - C = \emptyset$, so we don't have $A \succeq C$.