Let’s define a relation $T$ between natural numbers follows:

$aTb$ if and only if $a = b + 2k$, where $k$ is a natural number

Working directly from this definition, prove that $T$ is antisymmetric.
A closed interval of the real line can be represented as a pair \((c, r)\), where \(c\) is the center of the interval and \(r\) is its radius. Let \(X = \{(c, r) \mid c, r \in \mathbb{R}, r \ge 0\}\) be the set of closed intervals represented this way.

Now, let’s define the interval containment \(\preceq\) on \(X\) as follows

\[(c, r) \preceq (d, q) \text{ if and only if } r \le q \text{ and } |c - d| + r \le q.\]

Prove that \(\preceq\) is transitive.
Suppose that $T$ is a relation on the integers which is transitive. Let’s define a relation $R$ on the integers as follows:

$xRy$ if and only if there is an integer $k$ such that $xTk$ and $kTy$.

Prove that $R$ is transitive.
Let $T$ be the relation defined on $\mathbb{Z}^2$ by

$$(x, y)T(p, q) \text{ if and only if } x < p \text{ or } (x = p \text{ and } y \leq q)$$

Prove that $T$ is antisymmetric.
Define the relation $\sim$ on $\mathbb{Z}$ by

$$x \sim y \text{ if and only if } 5 \mid (3x + 7y)$$

Working directly from the definition of divides, prove that $\sim$ is transitive.
Let $A = \{(x, y, z) \in \mathbb{Z}^3 : x \leq y \leq z\}$. Let’s define a relation $R$ on $A$ as follows:

$$(a, b, c)R(x, y, z) \text{ if and only if } a \leq x \text{ and } z \leq b.$$ 

Working directly from this definition, prove that $R$ is antisymmetric.
Let \( A = \mathbb{Z}^+ \times \mathbb{Z}^+ \), i.e. pairs of positive integers.

Define a relation \( \gg \) on \( A \) as follows:

\[(x, y) \gg (p, q) \text{ if and only if there exists an integer } n \geq 1 \text{ such that } (x, y) = (np, nq).\]

Prove that \( \gg \) is antisymmetric.
Let \( A = \{(x, y, z) \in \mathbb{Z}^3 : x \leq y \leq z\} \). Let’s define a relation \( R \) on \( A \) as follows:

\[(a, b, c)R(x, y, z) \text{ if and only if } a \leq x \text{ and } z \leq b.\]

Working directly from this definition, prove that \( R \) is transitive.