1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it’s true.

For any sets $A$ and $B$, $(A \cap B) \cup (A \cap \overline{B}) = A$.

**Solution:** This claim is true. If $x$ is an element of $A$, there are exactly two possibilities: either $x$ is in $B$ or $x$ is not in $B$ (i.e. $x$ is in $\overline{B}$).

2. (4 points) Check the (single) box that best characterizes each item.

   - If $x \in A \cap B$, then $x \in A$. 
     - true for all sets $A$ and $B$  
     - true for some sets $A$ and $B$  
     - false for all sets $A$ and $B$  

   For all positive integers $n$, if $n! < -10$, then $n > 8$.
   - true  
   - false  
   - undefined

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   **Solution:**
   
1. (4 points) \( A = \{4, 5, 9\} \quad B = \{\text{arya}, \text{bran}\} \quad C = \{2, 4, 10\} \)

\[(A \cap C) \times B = \]

Solution: \(\{4\} \times B = \{(4, \text{arya}), (4, \text{bran})\}\)

\[|A \times B \times C| = \]

Solution: \(3 \times 2 \times 3 = 18\)

2. (4 points) Check the (single) box that best characterizes each item.

\[ A \times A = A \]

true for all sets A \[\square\] false for all sets A \[\checkmark\]

(Assume \(A \neq \emptyset\))

true for some sets A \[\square\]

\[\emptyset \subseteq A \]

true for all sets A \[\checkmark\] true for some sets A \[\square\]

false for all sets A \[\square\]

3. (7 points) In \(\mathbb{Z}_{11}\), find the value of \([6]^{42}\). You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as \([n]\), where \(0 \leq n \leq 10\).

Solution:

\([6]^2 = [36] = [3]\)
\([6]^8 = [9]^2 = [81] = [4]\)
1. (4 points) \( M = \{ \text{cereal, toast} \} \quad N = \{ \text{milk, coffee, wine} \} \)
\[ P = \{ \text{wine, beer, (coffee, ham), (milk, ham)} \} \]
\[ M \times (N - P) = \]
**Solution:** \( M \times (N - P) = M \times \{ \text{milk, coffee} \} \)
\[ = \{ (\text{cereal, milk}), (\text{cereal, coffee}), (\text{toast, milk}), (\text{toast, coffee}) \} \]
\|M \times N \times P| =
**Solution:** \( |M \times N \times P| = 2 \cdot 3 \cdot 4 = 24 \)

2. (4 points) Check the (single) box that best characterizes each item.

\( A \cup B = A \cap B \quad \checkmark \quad \text{true for all sets } A \text{ and } B \)
\( \quad \text{false for all sets } A \text{ and } B \quad \square \)

\( \{\emptyset\} \times \{\emptyset\} = \)
\[ \emptyset \quad \square \quad \{\emptyset\} \quad \square \quad \{\emptyset, \emptyset\} \quad \square \quad \{(\emptyset, \emptyset)\} \quad \checkmark \]

3. (7 points) In \( \mathbb{Z}_{17} \), find the value of \( [5]^{37} \). You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as \([n]\), where \(0 \leq n \leq 16\).

**Solution:**
So
1. (4 points) \( A = \{ \text{trump, rubio} \} \quad B = \{ \text{clinton, sanders} \} \)
\[ C = \{ (\text{trump, clinton}), (\text{sanders, rubio}) \} \]
\[ (B \times A) - C = \]
Solution: \( \{ (\text{clinton, trump}), (\text{clinton, rubio}), (\text{sanders, trump}) \} \)
\[ (A \cap C) \times B = \]
Solution: \( \emptyset \times B = \emptyset \)

2. (4 points) Check the (single) box that best characterizes each item.

\[ A \cap B = A \cup B \]
true for all sets \( A \) and \( B \) \( \Box \)
true for some sets \( A \) and \( B \) \( \Box \)
false for all sets \( A \) and \( B \) \( \Box \)

For all reals \( n \), if \( n^2 = 101 \),
then \( n > 11 \).
true \( \Box \)
false \( \Box \)
undefined \( \Box \)

3. (7 points) In \( \mathbb{Z}_9 \), find the value of \([4]^6 \times [5]^{20}\). You must show your work, keeping all numbers in your calculations small. You may not use a calculator. You must express your final answer as \([n]\), where \(0 \leq n \leq 8\).
Solution: \([5]^2 = [25] = [7]\)
\([5]^4 = [7]^2 = [49] = [4]\)
\([5]^8 = [4]^2 = [16] = [7]\)
\([5]^{16} = [7]^2 = [49] = [4]\)
\([4]^2 = [16] = [7]\)
\([4]^4 = [49] = [4]\)
1. (4 points) \( A = \{ \text{ginger, clove, nutmeg} \} \quad B = \{ \text{ginger, vanilla, pepper} \} \quad C = \{ \text{(clove, nutmeg)} \} \)

\( A \cap B = \) 

Solution: \( \{ \text{ginger} \} \)

\( A \cap C = \) 

Solution: \( \emptyset \)

2. (4 points) Check the (single) box that best characterizes each item.

For any sets \( A \) and \( B \), if \( x \in A - B \), then \( x \in A \).

- true \( \square \)
- false \( \square \)

\( \emptyset \subseteq A \)

- true for all sets \( A \) \( \square \)
- true for some sets \( A \) \( \square \)

- false for all sets \( A \) \( \square \)

3. (7 points) In \( \mathbb{Z}_{17} \), find the value of \([5]^{42}\). You must show your work, keeping all numbers in your calculations small. You may not use a calculator. You must express your final answer as \([n]\), where \( 0 \leq n \leq 16 \).

Solution:

\([5]^2 = [25] = [8]\)
\([5]^4 = [8]^2 = [64] = [-4]\)
\([5]^8 = [-4]^2 = [16] = [-1]\)
\([5]^{16} = [-1]^2 = [1]\)
\([5]^{32} = [1]^2 = [1]\)

So
1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it’s true.

For any sets $A$ and $B$, $A \cup (B - A) = A \cup B$.

**Solution:** This claim is true. If $x$ is in $A$, $x$ is clearly in both sets. So consider an $x$ that isn’t in $A$. If $x$ is in $A \cup (B - A)$ then $x \in (B - A)$, so $x$ is in $B$. Going the other way, if $x$ is in $A \cup B$ but not in $A$, then $x$ is in $B$ but not in $A$, so $x$ is in $B - A$.

2. (4 points) Check the (single) box that best characterizes each item.

- Let $A$ and $B$ be disjoint. $|A - B| = |A| - |B|$  
  true for all sets $A$ and $B$  
  false for all sets $A$ and $B$  
  true for some sets $A$ and $B$  
  $\square$

- $\{1, 2\} \cap \emptyset = \emptyset$  
  $\square$  
  $\{(1, \emptyset), (2, \emptyset)\}$  
  $\square$  
  $\{1, 2, \emptyset\}$  
  $\square$  
  $\{1, 2\}$  
  $\square$  
  undefined  
  $\square$

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

**Solution:** $[3]^2 = [9] = [2]$


1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it’s true.

For any sets \( A, B, \) and \( C \), if \( A \cap B = \emptyset \) and \( B \cap C = \emptyset \) then \( A \cap C = \emptyset \).

Solution: This claim is false. Consider \( A = C = \{1\} \) and \( B = \{2\} \). Then \( A \cap B = \emptyset \) and \( B \cap C = \emptyset \), but \( A \cap C = \{1\} \neq \emptyset \).

2. (4 points) Check the (single) box that best characterizes each item.

\(|A \cup B| \leq |A| + |B|\)  true for all sets \( A \) and \( B \)  \(\surd\)  true for some sets \( A \) and \( B \)  
false for all sets \( A \) and \( B \)  

\(\forall x \in \mathbb{Q}, \) if \( x^2 = 3 \), then \( x > 1000 \).  true  \(\surd\)  false  
undefined  

3. (7 points) In \( \mathbb{Z}_{13} \), find the value of \([7]^{19}\). You must show your work, keeping all numbers in your calculations small. You may not use a calculator. You must express your final answer as \([n]\), where \(0 \leq n \leq 12\).

Solution:
\([7]^2 = [49] = [10]\)
\([7]^4 = [100] = [9]\)
\([7]^8 = [9]^2 = [81] = [3]\)
So \([7]^{19} = [6]\)
1. (4 points)  \( A = \{\text{oak, apple, maple, elm}\} \quad B = \{\text{tree, leaf, oak}\} \quad C = \{(\text{oak, tree})\} \)

\[ |A \times (B - C)| = \]

**Solution:** \( (B - C) = B \). So \(|A \times (B - C)| = 4 \times 3 = 12\)

\( A \cap B = \)

**Solution:** \( A \cap B = \{\text{oak}\} \)

2. (4 points) Check the (single) box that best characterizes each item.

Sets \( A \) and \( B \) are disjoint

- \( A - B = B - A \)
- \( A \cap B = \{\emptyset\} \)
- \( A \cap B = \emptyset \)

\( \{1, 2\} \times \emptyset = \)

- \( \emptyset \)
- \( \{(1, \emptyset), (2, \emptyset)\} \)
- \( \{1, 2, \emptyset\} \)

\( \{0\} \times \emptyset = \)

- \( \emptyset \)
- \( \{1, 2\} \)

\( \{1, 2\} \times \emptyset = \)

- \( \emptyset \)
- \( \{(1, \emptyset), (2, \emptyset)\} \)
- \( \{1, 2, \emptyset\} \)

- undefined

3. (7 points) In \( \mathbb{Z}_{13} \), find the value of \([7]^{21}\). You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as \([n]\), where \(0 \leq n \leq 12\).

**Solution:**