CS 173, Spring 19 Examlet 3, white sheet

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A and B, $(A \cap B) \cup (A \cap \overline{B}) = A$.

   Solution: This claim is true. If x is an element of A, there are exactly two possibilities: either x is in B or x is not in B (i.e. x is in $\overline{B}$).

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cap B$, then $x \in A$.
   true for all sets A and B [√] true for some sets A and B [ ]
   false for all sets A and B [ ]

   For all positive integers n, if $n! < -10$, then $n > 8$.
   true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{37} = [3]^{32} \cdot [3]^4 \cdot [3] = [2][4][3] = [24] = [3]$

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{4, 5, 9\}$ $B = \{\text{arya}, \text{bran}\}$ $C = \{\ 2, 4, 10\ \}$

   $(A \cap C) \times B =$

   Solution: $\{4\} \times B = \{(4, \text{arya}), (4, \text{bran})\}$

   $|A \times B \times C| =$

   Solution: $3 \times 2 \times 3 = 18$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times A = A$ (Assume $A \neq \emptyset$)

   - true for all sets A ☐
   - false for all sets A ☑ (√)
   - true for some sets A ☐

   $\emptyset \subseteq A$

   - true for all sets A ☑ (√)
   - true for some sets A ☐
   - false for all sets A ☐

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^4 = [3]^2 = [9]$

   $[6]^8 = [9]^2 = [81] = [4]$

   $[6]^{16} = [4]^2 = [16] = [5]$

   $[6]^{32} = [5]^2 = [25] = [3]$

   $[6]^{42} = [6]^{32} \cdot [6]^8 \cdot [6]^2 = [3][4][3] = [36] = [3]$

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1. (4 points) $M = \{\text{cereal}, \text{toast}\}$ $N = \{\text{milk}, \text{coffee}, \text{wine}\}$
   $P = \{\text{wine}, \text{beer}, (\text{coffee}, \text{ham}), (\text{milk}, \text{ham})\}$

   $M \times (N - P) =$

   Solution: $M \times (N - P) = M \times \{\text{milk}, \text{coffee}\}$

   $= \{(\text{cereal}, \text{milk}), (\text{cereal}, \text{coffee}), (\text{toast}, \text{milk}), (\text{toast}, \text{coffee})\}$

   $|M \times N \times P| =$

   Solution: $|M \times N \times P| = 2 \cdot 3 \cdot 4 = 24$

2. (4 points) Check the (single) box that best characterizes each item.

   $\overline{A \cup B} = \overline{A} \cap \overline{B}$

   true for all sets A and B [✓] true for some sets A and B [ ]
   false for all sets A and B [ ]

   $\{\emptyset\} \times \{\emptyset\} =$

   $\emptyset$ [ ] $\{\emptyset\}$ [ ] $\{\emptyset, \emptyset\}$ [ ] $\{(\emptyset, \emptyset)\}$ [✓]

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [13] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{37} = [5]^{32} \cdot [5]^4 \cdot [5] = [1][-4][5] = [-20] = [14]$

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1. (4 points) $A = \{\text{trump}, \text{rubio}\}$ $B = \{\text{clinton}, \text{sanders}\}$

   $C = \{ (\text{trump}, \text{clinton}), (\text{sanders}, \text{rubio}) \}$

   $(B \times A) - C =$

   Solution: $\{(\text{clinton}, \text{trump}), (\text{clinton}, \text{rubio}), (\text{sanders}, \text{trump})\}$

   $(A \cap C) \times B =$

   Solution: $\emptyset \times B = \emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $A \cap B = A \cup B$

   - true for all sets A and B ☐
   - true for some sets A and B ☑ √
   - false for all sets A and B ☐

   For all reals n, if $n^2 = 101$, then $n > 11$.

   - true ☐
   - false ☑ √
   - undefined ☐

3. (7 points) In $\mathbb{Z}_9$, find the value of $[4]^6 \times [5]^{20}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[4]^2 = [16] = [7]$

   $[4]^4 = [49] = [4]$

   $[4]^6 \times [5]^{20} = [4]^4 \cdot [4]^2 \cdot [5]^{16} \cdot [5]^4 = [4] \cdot [7] \cdot [4] \cdot [4] = [28] \cdot [16] = [1] \cdot [7] = [7]$

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1. (4 points) $A = \{\text{ginger}, \text{clove}, \text{nutmeg}\}$ $B = \{\text{ginger}, \text{vanilla}, \text{pepper}\}$ $C = \{\ (\text{clove}, \text{nutmeg})\ \}$

   $A \cap B =$

   Solution: $\{\text{ginger}\}$

   $A \cap C =$

   Solution: $\emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   For any sets A and B, if $x \in A - B$, then $x \in A$. true [√] false [ ]

   $\{\emptyset\} \subseteq A$ true for all sets A [ ] true for some sets A [√] false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \le n \le 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{42} = [5]^{32} \cdot [5]^8 \cdot [5]^2 = [1][-1][8] = [-8] = [9]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A and B, $A \cup (B - A) = A \cup B$.

   Solution: This claim is true. If x is in A, x is clearly in both sets. So consider an x that isn't in A. If x is in $A \cup (B - A)$ then $x \in (B - A)$, so x is in B. Going the other way, if x is in $A \cup B$ but not in A, then x is in B but not in A, so x is in $B - A$.

2. (4 points) Check the (single) box that best characterizes each item.

   Let A and B be disjoint. $|A - B| = |A| - |B|$

   - true for all sets A and B ☐
   - true for some sets A and B ☑ √
   - false for all sets A and B ☐

   $\{1, 2\} \cap \emptyset =$

   - $\emptyset$ ☑ √
   - $\{(1, \emptyset), (2, \emptyset)\}$ ☐
   - $\{1, 2, \emptyset\}$ ☐
   - $\{\emptyset\}$ ☐
   - $\{1, 2\}$ ☐
   - undefined ☐

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{41} = [3]^{32} \cdot [3]^8 \cdot [3] = [2][2][3] = [12] = [5]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \cap B = \emptyset$ and $B \cap C = \emptyset$ then $A \cap C = \emptyset$.

   Solution: This claim is false. Consider $A = C = \{1\}$ and $B = \{2\}$. Then $A \cap B = \emptyset$ and $B \cap C = \emptyset$, but $A \cap C = \{1\} \neq \emptyset$.

2. (4 points) Check the (single) box that best characterizes each item.

   $|A \cup B| \leq |A| + |B|$ — true for all sets A and B [√] true for some sets A and B [ ] false for all sets A and B [ ]

   $\forall x \in \mathbb{Q}$, if $x^2 = 3$, then $x > 1000$. — true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^[2] \cdot [7] = [9] \cdot [10] \cdot [7]$

   $[9] \cdot [10] \cdot [7] = [90] \cdot [7] = [-1] \cdot [7] = [-7] = [6]$

   So $[7]^{19} = [6]$

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1. (4 points) $A = \{\text{oak}, \text{apple}, \text{maple}, \text{elm}\}$ $B = \{\text{tree}, \text{leaf}, \text{oak}\}$ $C = \{(\text{oak}, \text{tree})\}$

   $|A \times (B - C)| =$

   Solution: $(B - C) = B$. So $|A \times (B - C)| = 4 \times 3 = 12$

   $A \cap B =$

   Solution: $A \cap B = \{\text{oak}\}$

2. (4 points) Check the (single) box that best characterizes each item.

   Sets A and B are disjoint

   - $A - B = B - A$ ☐
   - $A = \overline{B}$ ☐
   - $A \cap B = \{\emptyset\}$ ☐
   - $A \cap B = \emptyset$ ☑ √

   $\{1, 2\} \times \emptyset =$

   - $\emptyset$ ☑ √
   - $\{(1, \emptyset), (2, \emptyset)\}$ ☐
   - $\{1, 2, \emptyset\}$ ☐
   - $\{\emptyset\}$ ☐
   - $\{1, 2\}$ ☐
   - undefined ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = ([7]^2)^2 = [-3]^2 = [9]$

   $[7]^8 = ([7]^4)^2 = [9]^2 = [81] = [3]$

   $[7]^{16} = ([7]^8)^2 = [3]^2 = [9]$

   $[7]^{21} = [7]^{16} \cdot [7]^4 \cdot [7] = [9] \cdot [9] \cdot [7] = [81] \cdot [7] = [3] \cdot [7] = [21] = [8]$