CS 173, Spring 19 Examlet 2, colored

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For all integers p and q, if $p^2(q^2 - 4)$ is odd, then p and q are odd.

You must begin by explicitly stating the contrapositive of the claim:

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(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

For all integers x, y, p, q and m, with $m > 0$, if $x \equiv p \pmod{m}$ and $y \equiv q \pmod{m}$, then $x^2 + xy \equiv p^2 + pq \pmod{m}$.

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(15 points) For any two real numbers x and y, the harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Using this definition and your best mathematical style, prove the following claim:

For any real numbers x and y, if $0 < x < y$, then $H(x, y) < y$.

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(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim by contrapositive.

> For all real numbers x and y, if x is not rational, then $2x + 3y$ is not rational or y is not rational.

You must begin by explicitly stating the contrapositive of the claim:

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(15 points) A triple (a, b, c) of positive integers is Pythagorean if $a^2 + b^2 = c^2$. Use proof by contrapositive to prove the following claim, using your best mathematical style and working directly from the definitions of "odd" and "even." (You may assume that odd and even are opposites.)

For any Pythagorean triple (a, b, c), if c^2 is odd, then a is even or b is even.

You must begin by explicitly stating the contrapositive of the claim:

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(15 points) For any two real numbers x and y, the harmonic mean is $H(x, y) = \frac{2xy}{x+y}$. Using this definition and your best mathematical style, prove the following claim:

For any real numbers x and y, if $0 < x < y$, then $x < H(x, y)$.

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(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $x \equiv y \pmod{k}$ if and only if $x = y + nk$ for some integer n.

For all integers a, b, c, p and k (c positive), if $ap \equiv b \pmod{c}$ and $k \mid a$ and $k \mid c$, then $k \mid b$.

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(15 points) Prove the following claim, using your best mathematical style. Hint: look at remainders and use proof by cases.

For any integer n, $n^2 + 2$ is not divisible by 4.