1. (9 points) Fill in key facts about the recursion tree for $T$, assuming that $n$ is even.

\[ T(0) = 5 \quad T(n) = 3T(n - 2) + n^2 \]

(a) The height:

(b) The number of leaves (please simplify):

(c) Value in each node at level $k$:

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that $f$ is to the left of $g$ if and only if $f(n) \ll g(n)$.

\[ n \quad n \log(17n) \quad \sqrt{n} + 18 \quad 8n^2 \quad 2^n + n! \quad 2^{\log_4 n} + 5^n \quad 0.001n^3 + 3^n \]

|   |   |   |   |   |   |   |   |
1. (7 points) Recall that \( f \) is \( O(g) \) if and only if there are positive reals \( c \) and \( k \) such that 
\[
0 \leq f(x) \leq cg(x)
\]
for every \( x \geq k \). Prof. Snape claims that there is a function \( f \) (from the reals to the reals) that can never be involved in a big-O relationship. Is he correct?

2. (8 points) Check the (single) box that best characterizes each item.

<table>
<thead>
<tr>
<th>Function</th>
<th>( \Theta(\log n) )</th>
<th>( \Theta(\sqrt{n}) )</th>
<th>( \Theta(n) )</th>
<th>( \Theta(n \log n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T(1) = c )</td>
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</tr>
</tbody>
</table>

\( n^{1.5} \) is
\[
\Theta(n^{1.614}) \quad O(n^{1.614}) \quad \text{neither of these}
\]

\( n^{\log_2 5} \) grows faster than \( n^2 \) \quad slower than \( n^2 \) at the same rate as \( n^2 \)
1. (9 points) Fill in key facts about the recursion tree for $T$, assuming that $n$ is even.

   $T(8) = 5 \quad T(n) = 3T(n - 2) + c$

   (a) The height:

   (b) The number of nodes at level $k$:

   (c) Value in each node at level $k$:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that $f$ is to the left of $g$ if and only if $f(n) \ll g(n)$.

   $3n^2 \quad \frac{n \log n}{7} \quad (10^{10})n \quad 0.001n^3 \quad 30 \log(n^{17}) \quad 8n! + 18 \quad 3^n + 11^n$
1. (7 points) Suppose that \( f \) and \( g \) are functions from the reals to the reals. Define precisely what it means for \( g \) to be \( \Theta(f) \). Your definition can be in terms of other primitives such as \( \ll \) and big-O.

2. (8 points) Check the (single) box that best characterizes each item.

\[
\begin{align*}
T(1) &= d \\
T(n) &= T(n-1) + c
\end{align*}
\]

\[
\begin{array}{cccccccc}
& \Theta(\log n) & \Theta(\sqrt{n}) & \Theta(n) & \Theta(n \log n) & \\
\Theta(n^2) & \Theta(n^3) & \Theta(2^n) & \Theta(3^n)
\end{array}
\]

\[
\begin{align*}
T(1) &= d \\
T(n) &= 2T(n/2) + c
\end{align*}
\]

\[
\begin{array}{cccccccc}
& \Theta(\log n) & \Theta(\sqrt{n}) & \Theta(n) & \Theta(n \log n) & \\
\Theta(n^2) & \Theta(n^3) & \Theta(2^n) & \Theta(3^n)
\end{array}
\]

Suppose \( f \) and \( g \) produce only positive outputs and \( f(n) \ll g(n) \). Will \( g(n) \) be \( O(f(n)) \)?

- no 
- sometimes 
- yes

\( n^{\log_2 4} \) grows faster than \( n^2 \)

\( n^{\log_2 4} \) grows slower than \( n^2 \)

\( n^{\log_2 4} \) grows at the same rate as \( n^2 \)
1. (9 points) Fill in key facts about the recursion tree for $T$, assuming that $n$ is a multiple of 3.

   $T(3) = 7 \quad T(n) = 2T(n - 3) + c$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Total work (sum of the nodes) at level $k$ (please simplify):

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that $f$ is to the left of $g$ if and only if $f(n) \ll g(n)$.

   $n \quad n \log(17n) \quad \sqrt{n} + 2^n + 18 \quad 8n^2 \quad 2^n + n! \quad 2^{\log_4 n} \quad 0.001n^3 + 3^n$


1. (7 points) Suppose that $f$, $g$, and $h$ are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

\[
\begin{align*}
T(1) &= c \\
T(n) &= 3T(n/3) + n \
\quad \Theta(\log n) &\quad \Theta(\sqrt{n}) &\quad \Theta(n) &\quad \Theta(n \log n) \\
\quad \Theta(n^2) &\quad \Theta(n^3) &\quad \Theta(2^n) &\quad \Theta(3^n)
\end{align*}
\]

\[
\begin{align*}
T(1) &= c \\
T(n) &= 2T(n/2) + n^2 \
\quad \Theta(\log n) &\quad \Theta(\sqrt{n}) &\quad \Theta(n) &\quad \Theta(n \log n) \\
\quad \Theta(n^2) &\quad \Theta(n^3) &\quad \Theta(2^n) &\quad \Theta(3^n)
\end{align*}
\]

Dividing a problem of size $n$ into $k$ sub-problems, each of size $n/m$, has the best big-$\Theta$ running time when

- $k < m$  
- $k = m$  
- $k > m$  
- $km = 1$

$n^{\log_2 5}$ grows faster than $n^2$  
$n^{\log_2 5}$ grows at the same rate as $n^2$  
$n^{\log_2 5}$ grows slower than $n^2$
1. (9 points) Fill in key facts about the recursion tree for $T$, assuming that $n$ is a power of 2.

\[ T(8) = 7 \quad T(n) = 4T\left(\frac{n}{2}\right) + n \]

(a) The height:

(b) Total work (sum of the nodes) at level $k$ (please simplify):

(c) The number of leaves (please simplify):

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that $f$ is to the left of $g$ if and only if $f(n) \ll g(n)$.

\[ 3^n \quad 4^{\log_2 n} \quad 2^{3n} \quad 3^{\log_4 4} \quad 0.1n \quad (5n)! \quad \sqrt{n} \]
1. (7 points) Suppose that $f$ and $g$ are functions from the reals to the reals. Define precisely when $f \preccurlyeq g$.

2. (8 points) Check the (single) box that best characterizes each item.

\[
\begin{align*}
T(1) &= d \\
T(n) &= 3T(n - 1) + c
\end{align*}
\]

\[
\begin{array}{cccccc}
\Theta(\log n) & \Theta(\sqrt{n}) & \Theta(n) & \Theta(n \log n) \\
\Theta(n^2) & \Theta(n^3) & \Theta(2^n) & \Theta(3^n)
\end{array}
\]

\[
\begin{align*}
T(1) &= d \\
T(n) &= T(n/2) + n
\end{align*}
\]

\[
\begin{array}{cccccc}
\Theta(\log n) & \Theta(\sqrt{n}) & \Theta(n) & \Theta(n \log n) \\
\Theta(n^2) & \Theta(n^3) & \Theta(2^n) & \Theta(3^n)
\end{array}
\]

\[
\begin{align*}
3^n \text{ is} & \\
3^n \text{ is}
\end{align*}
\]

\[
\begin{array}{cccc}
\Theta(5^n) & O(5^n) & \text{neither of these} \\
\Theta(2^n) & O(2^n) & \text{neither of these}
\end{array}
\]