CS 173 (B), Spring 2015, Examlet 6, Part B

NAME:	NETID:

Discussion Section: BDA:1PM BDB:2PM BDC:3PM BDD:4PM BDE:5PM

1. In how many ways can 10 students organize themselves into two study groups of 5 students each? Mark all the choices below that are equivalent to this. [10 points]

☑ A. Number of ways in which 5 black and 5 white chairs can be arranged in a row. ⟅3 for selecting⟆

☐ B. Number of partitions of a set of size 10 into exactly two parts. ⟅1 for not selecting⟆

☐ C. Number of ways to distribute 10 indistinguishable balls into two distinguishable bins, with 5 balls in each bin. ⟅1 for not selecting⟆

☑ D. Number of subsets of size 5 that a set of size 10 has. ⟅3 for selecting⟆

☐ E. Number of edges in the graph K_{10}. ⟅1 for not selecting⟆

☐ F. $10!/5!$ ⟅1 for not selecting⟆

2. Choose all the correct statements: [4 points]

⟅1 point for correctly marking/not marking each item.⟆

☑ A. $\binom{n}{3} = O(n^3)$

☐ B. $n! = O(2^n)$

☐ C. if $f(n) = \Theta(n)$ then $\exists c > 0,\ \lim_{n\to\infty} \frac{f(n)}{n} = c$

☑ D. if $\exists c > 0,\ \lim_{n\to\infty} \frac{f(n)}{n} = c$ then $f(n) = \Theta(n)$

3. **The Little Gadget (Lidget)** [6 points]

Lidget is a tiny device with n bits of internal memory. The n bits in memory can be toggled using n little white buttons on the side. There is also a large red button on top of the device: on

pressing this button, the Lidget sorts the bit string in its memory. For instance, with $n = 8$, if the memory has the string 00100110, and then the red button is pressed, the memory contents will be changed to 00000111.

(a) What is the total number of different states the Lidget can be in, as a function of n? (The state of the Lidget corresponds to the bit-string in its memory):

☐ A. $\Theta(n)$

☐ B. $\Theta(n^2)$

☑ C. $\Theta(2^n)$

☐ D. $\Theta(n!)$

(b) What is the total number of different states the Lidget can be in, immediately after pressing the red button?

☑ A. $\Theta(n)$

☐ B. $\Theta(n^2)$

☐ C. $\Theta(2^n)$

☐ D. $\Theta(n!)$

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2. Choose all the correct statements: [4 points]

⟅1 point for correctly marking/not marking each item.⟆

☐ A. $n! = O(2^n)$

☑ B. $\binom{n}{4} = O(n^4)$

☑ C. if $\exists c > 0$, $\lim_{n\to\infty} \frac{f(n)}{n} = c$ then $f(n) = \Theta(n)$

☐ D. if $f(n) = \Theta(n)$ then $\exists c > 0$, $\lim_{n\to\infty} \frac{f(n)}{n} = c$

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