CS 173 (B), Spring 2015, Examlet 4, Part B

NAME:	NETID:

Discussion Section: BDA:1PM BDB:2PM BDC:3PM BDD:4PM BDE:5PM

1. **Edges of the Hypercube**

   Recall that Q_n stands for the n-dimensional hypercube. Let α_n denote the number of edges in Q_n. Answer the following questions.

   (a) Choose the correct recurrence relation defining α_n. [5 points]

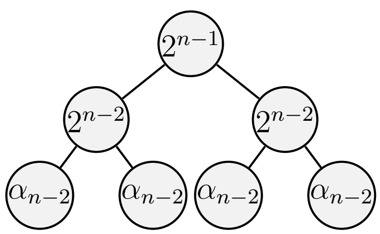
   ☐ A. $\alpha_0 = 1$ and $\alpha_n = 2\alpha_{n-1} + 2^n$ for $n \geq 1$ ≀2 points∫

   ☐ B. $\alpha_1 = 1$ and $\alpha_n = 2\alpha_{n-1}$ for $n \geq 2$

   ☐ C. $\alpha_1 = 0$ and $\alpha_n = 2\alpha_{n-1} + 2^n$ for $n \geq 2$ ≀1 point∫

   ☑ D. $\alpha_0 = 0$ and $\alpha_n = 2\alpha_{n-1} + 2^{n-1}$ for $n \geq 1$

   (b) Draw three levels of a recurrence tree for α_n, with α_{n-2} at the leaves. [5 points]



   ≀Full points if one more level shown (with α_{n-3} at the leaves). If 2^n in the root and 2^{n-1} in its children: 4 points, if A/C chosen above, 2 points otherwise. 1 point for some resemblance to the correct answer.∫

   (c) The closed-form formula for α_n is $\underline{\quad n \cdot 2^{n-1} \quad}$. [5 points]

   ≀$n \cdot 2^n \rightarrow$ if A/C chosen in part (a), then 4 points, else 2 points. 4 points for $(n \pm 1)2^{n-1}$. Also 4 points if consistent with the recursion tree above. 1 point if wrong but some resemblance to the correct answer.∫

2. How many nodes are there in a full-binary tree of depth d, if every internal node of the tree has at least one leaf node as a child? [5 points]

☐ A. $2^{d+1} - 1$

☐ B. $2^d + 1$

☑ C. $2d + 1$

☐ D. $d + 1$ ≀2 points∫

☐ E. There is not enough information to uniquely determine an answer.

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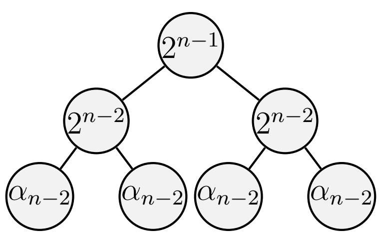

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