CS 173 (B), Spring 2015, Examlet 4, Part B

NAME:	NETID:

Discussion Section: BDA:1PM BDB:2PM BDC:3PM BDD:4PM BDE:5PM

1. **Regular Graphs.** [6 points]

   Recall that we say that a graph is d-regular if every node in the graph has degree exactly d. If G is 5-regular, then G has at least ___6___ nodes and ___15___ edges (and there is a 5-regular graph with that many nodes and edges).

2. **Chromatic Number.** [8 points]

   Consider the graphs G_1 and G_2 represented by the following adjacency matrices.

$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$G_1 \qquad\qquad G_2$$

   Then, $\chi(G_1) =$ ___3___ and $\chi(G_2) =$ ___2___.

3. **Induction.** [6 points]

   Suppose that the following claims have been proven regarding some predicate P defined over all integers.

   - $P(0)$ is true, $P(1)$ is false and $P(2)$ is false.
   - For all integers k, $P(k)$ is true if and only if $P(k+3)$ is true.

   Then what is the most that we can say about P? (Select one.)

- [ ] A. $\forall n \in \mathbb{N}$, $n \equiv 0 \pmod 3 \rightarrow P(n)$
- [ ] B. $\forall n \in \mathbb{Z}$, $n \equiv 0 \pmod 3 \rightarrow P(n)$
- [ ] C. $\forall n \in \mathbb{N}$, $n \equiv 0 \pmod 3 \leftrightarrow P(n)$
- [x] D. $\forall n \in \mathbb{Z}$, $n \equiv 0 \pmod 3 \leftrightarrow P(n)$
- [ ] E. None of the above is necessarily true.

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1. **Regular Graphs.** [6 points]

   Recall that we say that a graph is d-regular if every node in the graph has degree exactly d. If G is 5-regular, then G has at least ___6___ nodes and ___15___ edges (and there is a 5-regular graph with that many nodes and edges).

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$$G_1 \qquad\qquad G_2$$

   Then, $\chi(G_1) =$ ___2___ and $\chi(G_2) =$ ___4___.

3. **Induction.** [6 points]

   Suppose that the following claims have been proven regarding some predicate P defined over all integers.

   - $P(1)$ is true, $P(2)$ is false and $P(3)$ is false.
   - For all integers k, $P(k)$ is true if and only if $P(k+3)$ is true.

   Then what is the most that we can say about P? (Select one.)

☐ A. $\forall n \in \mathbb{Z}^+, n \equiv 1 \pmod 3 \rightarrow P(n)$

☐ B. $\forall n \in \mathbb{Z}, n \equiv 1 \pmod 3 \rightarrow P(n)$

☐ C. $\forall n \in \mathbb{Z}^+, n \equiv 1 \pmod 3 \leftrightarrow P(n)$

☑ D. $\forall n \in \mathbb{Z}, n \equiv 1 \pmod 3 \leftrightarrow P(n)$

☐ E. None of the above is necessarily true.