CS 173 (B), Spring 2015, Examlet 3, Part B

NAME:	**NETID:**

Discussion Section: BDA:1PM BDB:2PM BDC:3PM BDD:4PM BDE:5PM

1. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} x+1 & \text{if } x < -1 \\ x-1 & \text{if } x > 1 \\ 0 & \text{otherwise.} \end{cases} \qquad g(x) = \begin{cases} x-1 & \text{if } x < 0 \\ x+1 & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Mark all the correct choices below: [8 points]

- ☐ A. f is one-to-one.
- ☑ B. g is one-to-one.
- ☑ C. f is onto.
- ☐ D. g is onto.

♠ Treat as 4 True/False problems worth 2 point each.

2. For f and g as defined above, define $g \circ f$ and $f \circ g$. [8 points]

Solution:

$$g \circ f(x) = \begin{cases} x & \text{if } x < -1 \text{ or } x > 1 \\ 0 & \text{otherwise.} \end{cases} \qquad f \circ g(x) = x \text{ for all } x \in \mathbb{R}.$$

(Note that f is *an* inverse of g. On the other hand, f is not one-to-one and hence cannot have an inverse.)

♠ 4 points for each part. Full points for describing the functions alternatively (e.g., $f \circ g$ is the identity function, or just $f \circ g(x) = x$ without quantifying $\forall x \in \mathbb{R}$).
♠ -1 if $g \circ f$ is given as identity.
♠ Up to 2 points for each part, involving some elements of the correct answer.

3. Given relations $\sqsubset_1$ and $\sqsubset_2$ over a set S, define a new relation $\sqsubset_{12}$ over S as follows:

$\forall a, b \in S$, $a \sqsubset_{12} b$ if (and only if) $a \sqsubset_1 b$ or $a \sqsubset_2 b$.

Mark all the correct choices below: [4 points]

☑ A. If $\sqsubset_1$ is reflexive, so is $\sqsubset_{12}$.

☐ B. If $\sqsubset_1$ is irreflexive, so is $\sqsubset_{12}$. (Both need to be irreflexive for $\sqsubset_{12}$ to be so.)

☑ C. If both $\sqsubset_1$ and $\sqsubset_2$ are symmetric, so is $\sqsubset_{12}$.

☐ D. If both $\sqsubset_1$ and $\sqsubset_2$ are anti-symmetric, so is $\sqsubset_{12}$. (Consider having $a \sqsubset_1 b$ and $b \sqsubset_2 a$.)

♠ Treat as 4 True/False problems worth 1 point each.

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1. Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} x-1 & \text{if } x < 0 \\ x+1 & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases} \qquad g(x) = \begin{cases} x+1 & \text{if } x < -1 \\ x-1 & \text{if } x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

   Mark all the correct choices below: [8 points]

- ☑ A. f is one-to-one.
- ☐ B. g is one-to-one.
- ☐ C. f is onto.
- ☑ D. g is onto.

♠ Treat as 4 True/False problems worth 2 point each.

2. For f and g as defined above, define $g \circ f$ and $f \circ g$. [8 points]

$$g \circ f(x) = x \text{ for all } x \in \mathbb{R}. \qquad f \circ g(x) = \begin{cases} x & \text{if } x < -1 \text{ or } x > 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (Note that g is *an* inverse of f. On the other hand, g is not one-to-one and hence cannot have an inverse.)

♠ 4 points for each part. Full points for describing the functions alternatively (e.g., $g \circ f$ is the identity function, or just $g \circ f(x) = x$ without quantifying $\forall x \in \mathbb{R}$).
♠ -1 if $f \circ g$ is given as identity.
♠ Up to 2 points for each part, involving some elements of the correct answer.

3. Given relations $\sqsubset_1$ and $\sqsubset_2$ over a set S, define a new relation $\sqsubset_{12}$ over S as follows:
   $\forall a, b \in S$, $a \sqsubset_{12} b$ if (and only if) $a \sqsubset_1 b$ or $a \sqsubset_2 b$.
   Mark all the correct choices below: [4 points]

- ☑ A. If both $\sqsubset_1$ and $\sqsubset_2$ are symmetric, so is $\sqsubset_{12}$.
- ☐ B. If both $\sqsubset_1$ and $\sqsubset_2$ are anti-symmetric, so is $\sqsubset_{12}$. (Consider having $a \sqsubset_1 b$ and $b \sqsubset_2 a$.)
- ☑ C. If $\sqsubset_1$ is reflexive, so is $\sqsubset_{12}$.
- ☐ D. If $\sqsubset_1$ is irreflexive, so is $\sqsubset_{12}$. (Both need to be irreflexive for $\sqsubset_{12}$ to be so.)

♠ Treat as 4 True/False problems worth 1 point each.