CS 173 (B), Spring 2015, Examlet 2, Part B

NAME:	**NETID:**

Discussion Section: BDA:1PM BDB:2PM BDC:3PM BDD:4PM BDE:5PM

1. Consider using $\mathbb{Z}_2$ to model boolean logic. For this we represent T by 1 and F by 0. Let n_p stand for the element in $\mathbb{Z}_2$ that represents the proposition p. (Thus, if $p \equiv T$, then $n_p = 1$.) Then we can identify logical operations with arithmetic operations in $\mathbb{Z}_2$. For example, we have $n_{\neg p} = 1 + n_p$ (where + represents addition in $\mathbb{Z}_2$).

   Choose all of the correct formulas below, that hold for any propositions p and q. [10 points]

☑ A. $n_{p\wedge q} = n_p \cdot n_q$

☐ B. $n_{p\vee q} = n_p + n_q$

☑ C. $n_{\neg(p\vee q)} = (1 + n_p) \cdot (1 + n_q)$

☑ D. $n_{p\to q} = 1 + n_p + n_p \cdot n_q$

☑ E. $n_{p\oplus q} = n_p + n_q$

2. Consider two clocks with a "period" of m seconds and n seconds respectively. That is, after every m seconds, the needle of the first clock returns to its starting position; similarly, for the second clock, it takes n seconds before the needle returns to the starting position.

   Suppose you start off both clocks with their needles at their respective starting positions (pointing vertically up). How many seconds does it take before both the needles are simultaneously at the starting positions again? [5 points]

   $\mathrm{lcm}(m, n)$

3. In a certain year, January 1 was a Friday. This was a leap year (so 366 days in all). What day was December 31 of that year? [5 points]

   (Hint: $7 \times 52 = 364$.)

   Saturday

   ($1 \equiv 365 \pmod 7$), so the 365^{th} day is also a Friday. Dec 31 is the 366^{th} day.)

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   Choose all of the correct formulas below, that hold for any propositions p and q. [10 points]

- ☐ A. $n_{p \vee q} = n_p + n_q$
- ☑ B. $n_{p \wedge q} = n_p \cdot n_q$
- ☑ C. $n_{p \vee q} = 1 + (1 + n_p) \cdot (1 + n_q)$
- ☑ D. $n_{p \to q} = 1 + n_p + n_p \cdot n_q$
- ☐ E. $n_{p \leftrightarrow q} = n_p + n_q$

2. Consider two clocks with a "period" of m seconds and n seconds respectively. That is, after every m seconds, the needle of the first clock returns to its starting position; similarly, for the second clock, it takes n seconds before the needle returns to the starting position.

   Suppose you start off both clocks with their needles at their respective starting positions (pointing vertically up). How many seconds does it take before both the needles are simultaneously at the starting positions again? [5 points]

   $\text{lcm}(m, n)$

3. In a certain year, January 1 was a Saturday. This was not a leap year (so 365 days in all). What day was January 1 of the next year? [5 points]

   (Hint: $7 \times 52 = 364$.)

   Sunday

   ($1 \equiv 365 \pmod 7$, so the 365^{th} day is also a Saturday. Jan 1 next year is the 366^{th} day.)