CS 173, Fall 2013
Final Exam, A lecture

NAME:

NETID (e.g. hgrainger13):

Circle your discussion:

Th 2 **Th 3** **Th 4** **Th 5** **Fri 9** **Fri 10**

Problem	1	2	3a	3b	total
Possible	15	12	15	8	50
Score					

Total out of 50 points

You have 90 minutes to finish the exam.

We will be checking photo ID's during the exam. Have your ID handy.

Turn in your exam at the front when you are done.

Forgot your ID or discussion time? We have a photo roster.

INSTRUCTIONS (read carefully)

- There are 3 problems, each on a separate page. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and on the cover page table.
- Points may be deducted for solutions which are correct, but hard to read, hard to understand, poorly explained, or excessively complicated.
- When writing proofs, use your best mathematical style, e.g. introduce variables and assumptions, justify important or non-obvious steps, put your steps in logical order.
- Brief explanations and/or showing work, even when not explicitly asked for, may increase partial credit for buggy answers. Partial credit for multiple-choice questions is very rare.
- It is not necessary to simplify or calculate out complex constant expressions such as $(0.7)^3(0.3)^5$, $\frac{0.15}{3.75}$, 3^{17}, and $7!$, unless it is explicitly indicated to completely simplify.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam.
  Turn off your cell phone now.
  No notes or electronic devices of any kind are allowed.
  These should be secured in your bag and out of reach during the exam.
- Do all work in the space provided, using the backs of sheets if necessary.
  If your work is on the backside then you must clearly indicate so on the problem.
  See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, you may discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem 1: Multiple choice (15 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

If there is a one-to-one function from A to B, then we know that

- $|A| \leq |B|$ ☐
- $|B| \leq |A|$ ☐
- $|B| = |A|$ ☐
- none of the above ☐

$|\mathbb{P}(A)| = |A|$

- true ☐
- false ☐
- true if A is finite ☐

How many states (aka board configurations) does the game of chess have?

- finite ☐
- countably infinite ☐
- uncountable ☐

There are more functions from $\mathbb{N}$ to $\mathbb{N}$ than there are finite-length ASCII formulas.

- true ☐
- false ☐

$$\sum_{k=0}^{n} \binom{n}{k}$$

- n ☐
- $\binom{n+1}{k}$ ☐
- 2^n ☐
- n^2 ☐
- $(n+1)^k$ ☐
- $\binom{2n}{k} - 1$ ☐

Hint: what is $\binom{n}{k}$ the size of?

Problem 2: Multiple choice (12 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

$|\mathbb{Z} \times \mathbb{Z}| = |\mathbb{C}|$, where $\mathbb{C}$ is the set of complex numbers.

true ☐ false ☐ not known ☐

Building a general-purpose program that can decide if another program stops or runs forever.

possible ☐ impossible ☐ not known ☐

$f : \mathcal{P}(\mathbb{N}) \rightarrow \mathcal{P}(\mathbb{N} \times \mathbb{N})$.
Then $f(\emptyset)$ is

a pair of natural numbers ☐	0 ☐
a set of natural numbers ☐	(0,0) ☐
a pair of sets of natural numbers ☐	{(0,0)} ☐
a set of pairs of natural numbers ☐	

Two state diagrams are considered the same if they are the same as labelled graphs. That is, they have the same number of states, with the same structure of edges and labels on the edges. Let's suppose that our state diagrams all use the same finite set of actions A, and that each state diagram has a finite set of states named $1, \ldots, n$, where n can be as large as you like.

The number of different state diagrams is:

finite ☐ countably infinite ☐ uncountable ☐

Problem 3: State Diagrams (23 points)

Recall that a phone lattice is a state diagram representing a set of words. Each edge in a phone lattice has a single letter on it. In a "deterministic" state diagram, the transition function never returns more than one state. That is, it returns either the empty set or a set containing a single state. Said another way, if you look at any state s and any letter a, there is never more than one edge labelled a leaving state s.

(a) (15 points) Draw a phone lattice representing exactly the following set of words. Your phone lattice must be deterministic, have only one start state, and contain no more than 14 states. For full credit, avoid using more states than necessary.

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moodle, moon, doodle
moo, mooo, moooo, ...    [i.e. m followed by two or more o's]
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(b) (8 points) Suppose that we have an alphabet of p letters and a fixed set of n states. In how many different ways can we build a deterministic phone lattice using these states and letters? Briefly explain your answer or show work.