CS 173: Discrete Structures, Spring 2014
Exam 1 Review

These problems are to help you review for the first midterm. They should not be handed in.

1. **Euclidean algorithm**

   Trace the execution of the Euclidean algorithm for computing GCD on the inputs $a = 837$ and $b = 2015$. That is, give a table showing the values of the main variables (x, y, r) for each pass through the loop. Explicitly indicate what the output value is.

2. **Direct Proof Using Congruence mod k**

   In the book, you will find several equivalent ways to define congruence mod k. For this problem, use the following definition: for any integers x and y and any positive integer m, $x \equiv y \pmod{m}$ if there is an integer k such that $x = y + km$.

   Using this definition prove that, for all integers a, b, c, p, q where p and q are positive, if $a \equiv b \pmod{p}$ and $c \equiv b \pmod{q}$ and $q|p$, then $a - 2c \equiv (-b) \pmod{q}$.

3. **Equivalence classes**

   Let $A = \mathbb{R}^{\geq 0} \times \mathbb{R}^{\geq 0} - \{(0,0)\}$, i.e. pairs of non-negative reals in which no more than one of the two numbers is zero.

   Consider the equivalence relation $\sim$ on A defined by

   $$(x, y) \sim (p, q) \quad \text{iff} \quad (xy)(p + q) = (pq)(x + y)$$

   (a) List four elements of $[(3, 1)]$. Hint: what equation do you get if you set (x, y) to $(3, 1)$ and $q = 2p$?

   (b) Give two other distinct equivalence classes that are not equal to $[(3, 1)]$.

   (c) Describe the members of $[(0, 4)]$.

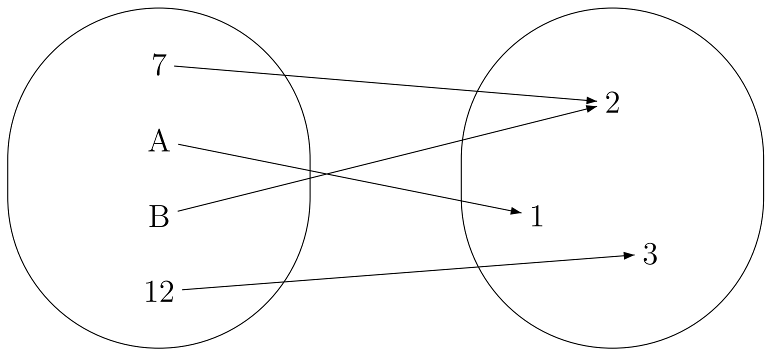
4. **Subset proof**

   Suppose that A, B and C are sets. Recall the definition of $X \subseteq Y$: for every p, if $p \in X$, then $p \in Y$. Prove that if $A \subseteq B$ then $A \cap C \subseteq B \cap C$. Briefly justify the key steps in your proof.

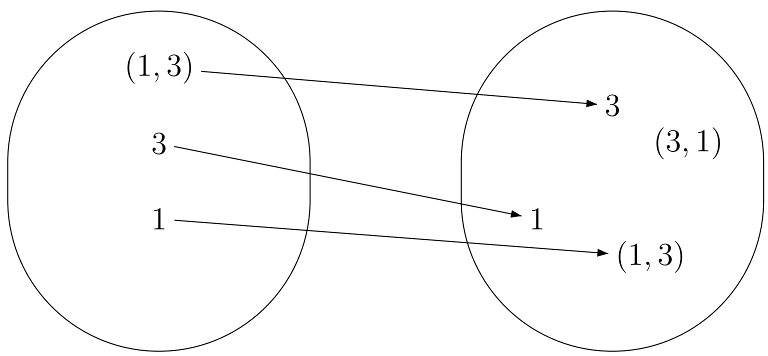
5. **Functions**

For each of the following functions determine if it is onto or not onto. Briefly, but clearly, justify your answers. (A full formal proof is not required.)

(a) The function f given by the following diagram where the left bubble represents the domain and the right the codomain:



(b) The function g given by the following diagram:



(c) $h : \mathbb{Z} \to \mathbb{Z}$ by $h(x) = 3\lceil \frac{x}{3} \rceil$

(d) $k : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ by $k(x, y) = x$

6. **One-to-one**

Which of these functions are one-to-one? Briefly justify your answers.

(a) $h : [0, 1] \to \mathbb{R}^2$ such that $h(\lambda) = \lambda(2, 2) + (1 - \lambda)(1, 3)$ where you use the following formula to multiply a real number a by a 2D point (x, y):

$$a(x, y) = (ax, ay)$$

(b) $f : \mathbb{N}^2 \to \mathbb{N}$ such that $f(x, y) = 4^x 3^y$

(c) $k : \mathbb{N} \times \mathbb{N} \to \mathbb{Z}$ such that $k(x, y) = (1 - x^2) \left\lfloor \frac{y}{3} \right\rfloor$