CS 173, Fall 2013
Midterm 1, 1 October 2013

NAME:

NETID (e.g. hpotter7):

Circle your discussion:

Th 2   Th 3   Th 4   Th 5   Fri 9   Fri 10

Fri 11   Fri 12   Fri 1   Fri 2   Fri 3   Fri 4

Problem	1	2	3a	3b	4a	4b	1-4
Possible	10	12	10	6	6	6	50
Score							

Problem	5a	5b	5c	6	7	5-7
Possible	6	6	6	16	16	50
Score						

Total   out of 100 points

You have 75 minutes to finish the exam.

We will be checking photo ID's during the exam. Have your ID handy.

Turn in your exam at the front when you are done.

Forgot your ID or discussion time? We have a photo roster.

INSTRUCTIONS (read carefully)

- There are 7 problems, each on a separate page. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and on the cover page table.
- Points may be deducted for solutions which are correct, but hard to read, hard to understand, poorly explained, or excessively complicated.
- When writing proofs, use your best mathematical style, e.g. introduce variables and assumptions, justify important or non-obvious steps, put your steps in logical order.
- Brief explanations and/or showing work, even when not explicitly asked for, may increase partial credit for buggy answers. Partial credit for multiple-choice questions is very rare.
- It is not necessary to simplify or calculate out complex constant expressions such as $(0.7)^3(0.3)^5$, $\frac{0.15}{3.75}$, 3^{17}, and $7!$, unless it is explicitly indicated to completely simplify.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam.
  Turn off your cell phone now.
  No notes or electronic devices of any kind are allowed.
  These should be secured in your bag and out of reach during the exam.
- Do all work in the space provided, using the backs of sheets if necessary.
  If your work is on the backside then you must clearly indicate so on the problem.
  See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, you may discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem 1: Multiple choice (10 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

$\lfloor x \rfloor < \lceil x \rceil$

- true for any real number x ☐
- false for any real number x ☐
- true for some real numbers x ☐

For any integer x, if $x \geq 8$, then $x^2 \geq 36$.

True ☐ False ☐

$p \rightarrow \neg q \equiv q \rightarrow \neg p$

True ☐ False ☐

$|A \cup B| = |B| + |A|$

- True for any sets A and B ☐
- False for any sets A and B ☐
- True for some sets A and B ☐

$\emptyset \in A$

- true for any set A ☐
- false for any set A ☐
- true for some sets A ☐

Problem 2: Multiple choice (12 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

$-3 \equiv 7 \pmod{10}$	True ☐	False ☐
For any positive integers a and b, $\gcd(a,b) = \gcd(b \bmod a, a)$	True ☐	False ☐
For any positive integers p and q, if $\text{lcm}(p,q) = pq$, then p and q are relatively prime.	True ☐	False ☐
A relation cannot be both symmetric and antisymmetric.	True ☐	False ☐

A linear order is a special type of partial order. What extra property does it have?

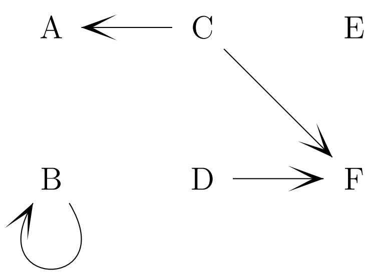
- all elements are reflexive ☐
- all pairs of elements are comparable ☐
- it is strict ☐
- it is homogeneous ☐

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then $\mathbb{R}$ is

- the domain of f ☐
- the co-domain of f ☐
- the image of f ☐

Problem 3: Short answer (16 points)

(a) (10 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$

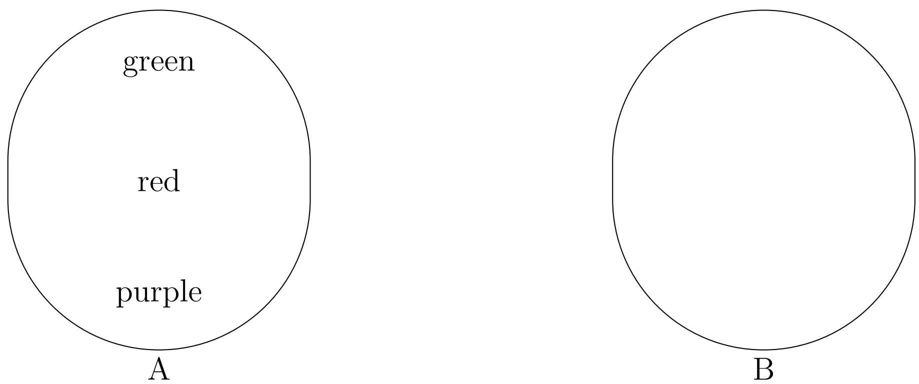


Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

(b) (6 points) The following picture shows the contents of a set A. Complete it to make an example of a function from A to B that is onto but not one-to-one. That is, add elements to set B and draw arrows between the two sets showing how input values map to output values. The values in B should be integers.



Problem 4: Short answer (12 points)

(a) (6 points) Let R be the relation on the real numbers such that xRy if and only if $\lfloor x/2 \rfloor = \lfloor y/2 \rfloor$. Describe which values are in [17]. (Do not simply wrap the definition in set-builder notation: make it clear that you understand what's in this set.)

(b) (6 points) In $\mathbb{Z}_{11}$, find the value of $[8]^{22}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

Problem 5: Short answer (18 points)

(a) (6 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be antisymmetric.

(b) (6 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

Claim: For any positive integers a, b, k, j, if $a \equiv b \pmod{k}$ and $k|j$, then $a \equiv b \pmod{j}$.

(c) (6 points) State the negation of the following claim. Your answer should be in words, with all negations (e.g. "not") on individual predicates.

For every cat k, if k is orange, then k is large or k is not friendly.

Problem 6: Relation Proof (16 points)

Let $A = \mathbb{R}^+ \times \mathbb{R}^+$, i.e A is the set of pairs of positive real numbers. Suppose that T is the relation on A such that $(a, b)T(p, q)$ if and only if $bp \geq aq$. Prove that T is transitive. Hint: it's ok to use division.

Write your netID, in case this page gets pulled off:

Problem 7: Set theory proof (16 points)

$$A = \{(p, q) \in \mathbb{Z}^2 \mid 2pq + 6q - 5p - 15 \geq 0\}$$
$$B = \{(s, t) \in \mathbb{Z}^2 \mid s \geq 0\}$$
$$C = \{(x, y) \in \mathbb{Z}^2 \mid y \geq 0\}$$

Prove that $(A \cap B) \subseteq C$.

Use your best mathematical style, e.g. introduce variables and assumptions, justify important or non-obvious steps, put your steps in logical order. You must use the method of selecting an element from the smaller set and showing that it belongs to the larger set.