CS 173, Spring 2014, B Lecture
Midterm 1, 25 February 2014

NAME:

NETID (e.g. hpotter23, not 314159265):

Circle your discussion:

Th 2   **Th 3**   **Th 4**   **Th 5**   **Fri 10**   **Fri 11**   **Fri 4**

Problem	1	2	3	4	Subtotal
Possible	15	16	12	14	57
Score					
Problem		5	6	7	Subtotal
Possible		12	15	16	43
Score					

Total   out of 100 points

We will be checking photo ID's during the exam. Have your ID handy.
(Forgot your ID? See us at the end of the exam.)

Turn in your exam at the front when you are done.

You have 75 minutes to finish the exam.

INSTRUCTIONS (read carefully)

- There are 7 problems, each on a separate page. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and on the cover page table.
- Points may be deducted for solutions which are correct, but hard to read, hard to understand, poorly explained, or excessively complicated.
- When writing proofs, use your best mathematical style, e.g. introduce variables and assumptions, justify important or non-obvious steps, put your steps in logical order.
- Brief explanations and/or showing work, even when not explicitly asked for, may increase partial credit for buggy answers. Partial credit for multiple-choice questions is very rare.
- It is not necessary to simplify or calculate out complex constant expressions such as $(0.7)^3(0.3)^5$, $\frac{0.15}{3.75}$, 3^{17}, and $7!$, unless it is explicitly indicated to completely simplify.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam.
  Turn off your cell phone now.
  No notes or electronic devices of any kind are allowed.
  These should be secured in your bag and out of reach during the exam.
- Do all work in the space provided, using the backs of sheets if necessary.
  If your work is on the backside then you must clearly indicate so on the problem.
  See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, you may discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem 1: Multiple choice (15 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

$\neg(p \to q) \equiv$

- $q \wedge \neg p$ ☐
- $\neg p \to \neg q$ ☐
- $q \vee \neg p$ ☐
- $\neg q \wedge p$ ☐

If x is odd, then $x \equiv 1 \pmod 2$

- always ☐
- sometimes ☐
- never ☐

$\forall x \in \mathbb{R},\ x^2 < 0 \ \to\ x < 0$

- true ☐
- undefined truth value ☐
- false ☐

$\forall k \in \mathbb{Z},\ 9k^2 + 2 \equiv 2 \pmod{3k}$

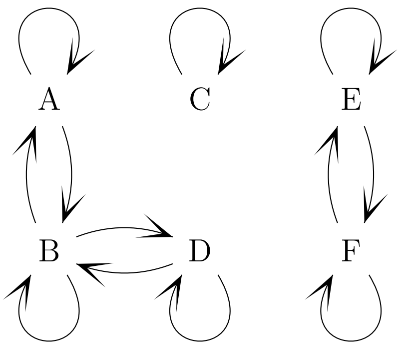
- true ☐
- false ☐

$\forall x, y \in \mathbb{Z}^+,\ lcm(x, x^2 y) =$

- x ☐
- $x^2 y$ ☐
- $x^2 \gcd(x, y)$ ☐
- $x^3 y$ ☐

Problem 2: Short answer (16 points)

(a) (10 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

(b) (6 points) Consider the function $f : \mathbb{N} \to \mathbb{Z}, f(x) = x^2$. Is f onto, one-to-one, both, or neither? Briefly explain why.

Problem 3: Short answer (12 points)

(a) (6 points) Let $f : \mathbb{N}^2 \to \mathbb{R}, f(x, y) = x + \frac{1}{y+5}$ and $g : \mathbb{R} \to \mathbb{R}, g(x) = 2x$. Write the expression for $(g \circ f)(x, y)$, and compute $(g \circ f)(3, 5)$. Show your work.

(b) (6 points) Suppose we have the following sets:

$$\begin{aligned} A &= \{a, b, c, d, e, ..., x, y, z\} = \{\text{all 26 lowercase letters}\} \\ B &= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \\ C &= \{2, 4\} \\ D &= \{5, 4, 7\} \end{aligned}$$

$|A \times B \times C| =$

$B - (C \cup D) =$

$A \cap B =$

Problem 4: Short answer (14 points)

(a) (6 points) Use the Euclidean algorithm to compute gcd(2262, 546)

(b) (8 points) In $\mathbb{Z}_7$, find the value of $[11]^{12}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

Problem 5: Relations and Logic (12 points)

(a) (6 points) Suppose that $Cats$ is the set of all cats. Let $S(x)$ mean that cat x has stripes, $L(x)$ that a cat has long fur, and $C(x)$ that a cat is cute. Write a logical expression for the **negation** of the following statement. Your final answer should use only shorthand notation (e.g. $\forall$ and $L(x)$) and all instances of $\neg$ must be on individual predicates.

All cats with long hair and stripes are cute.

(b) (6 points) Suppose that R is the relation on the set of integers such that aRb if and only if $\gcd(a, b) > 1$. Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Problem 6: Proof (15 points)

Let the set A be $\{(x, y) : (x, y) \in \mathbb{R}^2, |x| \geq 1 \text{ or } |y| \geq 1\}$

Let the set B be $\{(a, b) : (a, b) \in \mathbb{R}^2, b = 5 - a^2\}$

Prove that B $\subseteq$ A. Hint: you may find proof by cases helpful.

Write your netID, in case this page gets pulled off:

Problem 7: Proof (16 points)

Suppose that n is some integer ≥ 2. Let's define the relation R_n on the integers such that aR_nb if and only if $a \equiv b + 1 \pmod{n}$. Prove the following claim

> Claim: If R_n is symmetric, then $n = 2$.

You must work directly from the definition of congruence mod k, using the following version of the definition: $x \equiv y \pmod{k}$ iff $x - y = mk$ for some integer m. You may use the following fact about divisibility: for any non-zero integers p and q, if $p \mid q$, then $|p| \leq |q|$.