CS 173, Fall 2012
Midterm 1, 2 October 2012

NAME:

NETID (e.g. hpotter23, not 123987654):

DISCUSSION DAY:

DISCUSSION TIME:

You will lose a point if you don't accurately write the day and time of the discussion you are officially registered for. If you don't know the day and/or time of your discussion, you may consult the photo rosters at the podium before turning in your exam.
If you have recently changed section, check here: ☐

Problem	1	2	3	4	5	6	Total
Possible	10	13	12	13	10	12	70
Score							

We will be checking photo ID's during the exam. Have your ID handy.
(Forgot your ID? See us at the end of the exam.)

Turn in your exam at the front when you are done.

You have 75 minutes to finish the exam.

INSTRUCTIONS (read carefully)

- There are 6 problems, each on a separate page. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and on the cover page table.
- Points may be deducted for solutions which are correct, but hard to read, hard to understand, poorly explained, or excessively complicated.
- When writing proofs, use your best mathematical style, e.g. introduce variables and assumptions, justify important or non-obvious steps, put your steps in logical order.
- Brief explanations and/or showing work, even when not explicitly asked for, may increase partial credit for buggy answers. Partial credit for multiple-choice questions is very rare.
- It is not necessary to simplify or calculate out complex constant expressions such as $(0.7)^3(0.3)^5$, $\frac{0.15}{3.75}$, 3^{17}, and 7!, unless it is explicitly indicated to completely simplify.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam.
  Turn off your cell phone now.
  No notes or electronic devices of any kind are allowed.
  These should be secured in your bag and out of reach during the exam.
- Do all work in the space provided, using the backs of sheets if necessary.
  If your work is on the backside then you must clearly indicate so on the problem.
  See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, you may discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem 1: Multiple choice (10 points)

Check the most appropriate box for each statement. Check only one box per statement. If you change your answer, make sure it's easy to tell which box is your final selection.

$\sum_{k=3}^{n} k^7 =$

- $\sum_{p=1}^{n-2} (p+2)^7$ ☐
- $\sum_{p=1}^{n-2} p^9$ ☐
- $\sum_{p=1}^{n-2} k^7$ ☐
- $\sum_{p=1}^{n-2} k^9$ ☐

For all real numbers x,
if $x^2 \leq -3$, then $x < 10$

- **True** ☐
- **False** ☐

zero is

- even ☐
- odd ☐
- both ☐
- neither ☐

$\{4, 5, 7\} \cap \{7, 8, 9\} =$

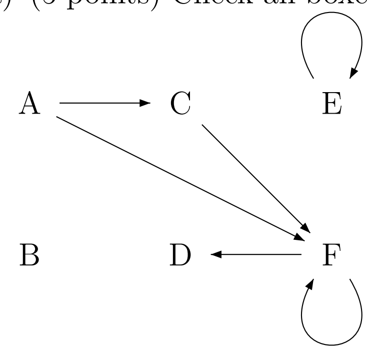
- 7 ☐
- $\{7\}$ ☐
- $\{4, 5, 8, 9\}$ ☐
- $\{4, 5, 7, 8, 9\}$ ☐

$|A \cup B| = |A| + |B|$

- true for any sets A and B ☐
- false for any sets A and B ☐
- true for some sets A and B ☐

Problem 2: Short answer (13 points)

(a) (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$



Reflexive: ☐ **Irreflexive**: ☐

Symmetric: ☐ **Antisymmetric**: ☐

Transitive: ☐

(b) (3 points) Suppose that $f : \mathbb{Z} \to \mathbb{R}$ is defined by $f(n) = 3n$. Identify clearly (e.g. $\mathbb{C}$, $\{$powers of two$\}$) the key sets in this definition.

domain:

co-domain:

image:

(c) (5 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

Problem 3: Number Theory (12 points)

(a) (6 points) In $\mathbb{Z}_{11}$, find the value of $[6]^6 + [5]^3$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

(b) (6 points) Let a and b be integers, $b > 0$. The "Division Algorithm" uses two formulas to define the quotient q and the remainder r of a divided by b. State these two formulas.

Problem 4: Sets (13 points)

(a) (8 points) Let $A = \{(x, y) \in \mathbb{R}^2 \mid y = \frac{2}{6}x - 3\}$ and $B = \{(x, y) \in \mathbb{R}^2 \mid x \geq 3y\}$. Prove that $A \subseteq B$.

(b) (5 points) Suppose we have the following sets:

$$\begin{aligned} M &= \{\text{cereal}, \text{toast}\} \\ N &= \{\text{milk}, \text{coffee}, \text{wine}, \text{juice}\} \\ P &= \{\text{wine}, \text{beer}, (\text{coffee}, \text{ham})\} \end{aligned}$$

List the elements of $M \times (N - P)$

Problem 5: Proofs/logic (10 points)

(a) (5 points) Suppose that J is the set of open intervals of the real line, i.e

$$J = \{(x, y) \in \mathbb{R}^2 \mid x < y\}$$

Let's define the "touches" relation T on J by $(a, b)T(c, d)$ if and only if $a = d$ or $b = c$.
Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

(b) (5 points) State the contrapositive of the following claim. Your answer should be in words, with all negations (e.g. "not") on individual predicates.

> For all hyperreal numbers x and y, if x is floppy and y is typical, then x is acidic or $x + y$ is bubbly.

Write your netID, in case this page gets pulled off:

Problem 6: Number Theory Proof (12 points)

Congruence mod k can be defined as follows: if a, b, k are integers, k positive, then $a \equiv b \pmod{k}$ if and only if $a = b + nk$ for some integer n. Using this definition and our normal definition of $m \mid n$, prove the following claim. Use your best mathematical style, e.g. introduce variables and assumptions, justify important or non-obvious steps, put your steps in logical order.

> Claim: For any integers a, b, c, k, q, where k and q are positive,
> if $a \equiv b \pmod{k}$ and $b \equiv c \pmod{q}$ and $k \mid q$, then $a \equiv c \pmod{k}$.