

CS 173: Discrete Structures, Spring 2013

Exam 1 Review

These problems are to help you review for the first midterm. They should not be handed in.

1. Set Operations

Suppose you were given the following sets:

$$\begin{aligned}\mathbf{A} &= \{\text{Piano}, (\text{Violin}, \text{Viola}, \text{Cello}), \text{Guitar}\} \\ \mathbf{B} &= \{(\text{Flute}, \text{Piccolo}), \text{Cymbals}\} \\ \mathbf{C} &= \{\text{Piano}, \text{Flute}\} \\ \mathbf{D} &= \{(\text{Violin}, \text{Viola}, \text{Cello}), (\text{Flute}, \text{Piccolo})\}\end{aligned}$$

List the elements of the set or find the values for the following expressions:

(a) $|A|$

(b) $A \cup D$

(c) $A \cap C$

(d) $B \cap C$

(e) $A - (B - C)$

(f) $(B \cap D) \times C$

(g) $A \times \emptyset$

(h) $C \times \{\emptyset\}$

2. Counting with sets

In our role-playing game, an evil character may be an elf or a troll, it may be red, green, brown, or black, and it may have scales or hair. A good character may be an elf or a human or a lion, it may be green, brown, or blue, and it has hair or fur. How many different character types do we have?

Suppose we have a 26 character alphabet. How many 6-letter strings start with PRE or end with TH?

3. Euclidean algorithm

Trace the execution of the Euclidean algorithm for computing GCD on the inputs $a = 837$ and $b = 2015$. That is, give a table showing the values of the main variables (x, y, r) for each pass through the loop. Explicitly indicate what the output value is.

4. Direct Proof Using Congruence mod k

In the book, you will find several equivalent ways to define congruence mod k . For this problem, use the following definition: for any integers x and y and any positive integer m , $x \equiv y \pmod{m}$ if there is an integer k such that $x = y + km$.

Using this definition prove that, for all integers a, b, c, p, q where p and q are positive, if $a \equiv b \pmod{p}$ and $c \equiv b \pmod{q}$ and $q|p$, then $a - 2c \equiv (-b) \pmod{q}$.

5. Equivalence classes

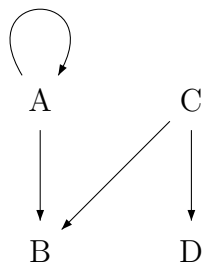
Let $A = \mathbb{R}^{\geq 0} \times \mathbb{R}^{\geq 0} - \{(0, 0)\}$, i.e. pairs of non-negative reals in which no more than one of the two numbers is zero.

Consider the equivalence relation $\sim$ on A defined by

$$(x, y) \sim (p, q) \quad \text{iff} \quad (xy)(p + q) = (pq)(x + y)$$

- (a) List four elements of $[(3, 1)]$. Hint: what equation do you get if you set (x, y) to $(3, 1)$ and $q = 2p$?
- (b) Give two other distinct equivalence classes that are not equal to $[(3, 1)]$.
- (c) Describe the members of $[(0, 4)]$.

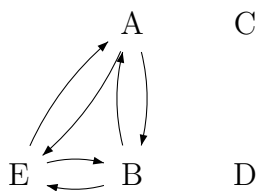
6. Relation properties



Reflexive: ☐ Irreflexive: ☐
 Symmetric: ☐ Antisymmetric: ☐
 Transitive: ☐

$\sim$ is the relation on $\mathbb{R}$ such that $x \sim y$ if and only if $xy = 1$

Reflexive: ☐ Irreflexive: ☐
 Symmetric: ☐ Antisymmetric: ☐
 Transitive: ☐



Reflexive: ☐ Irreflexive: ☐
 Symmetric: ☐ Antisymmetric: ☐
 Transitive: ☐

7. Proofs on Relations

- (a) Define a relation $\sim$ on the set of all functions from $\mathbb{R}$ to $\mathbb{R}$ by the rule $f \sim g$ if and only if $\exists k \in \mathbb{R}$ such that $f(x) = g(x)$ for every $x \geq k$. Prove that $\sim$ is an equivalence relation.