Proving an inequality by induction

Consider the recurrence defined as:

$$T(n) = \begin{cases} 1 & \text{if } n \leq 7 \\ 4T(\lfloor \frac{n}{2} \rfloor) + 7 & \text{if } n \geq 8 \end{cases}$$

Prove that $\forall n \geq 1, \ T(n) < 7n^2 - 3$

Proof: We prove this by induction on n.

Base case ($n = 1$): $T(1) = 1 < 4 = 7(1^2) - 3$. Hence the base case is true.

Strong Inductive Hypothesis: For some $k \geq 1$, $\forall 1 \leq m \leq k, \ T(m) < 7m^2 - 3$

Inductive step: We need to show that $T(k+1) < 7(k+1)^2 - 3$.

Case 1 ($k + 1 \leq 7$): Then $T(k+1) = 1 < 4 = 7(1^2) - 3 < 7(k+1)^2 - 3$, since $k + 1 > 1$.

Case 2 ($k + 1 \geq 8$): Now

$$\begin{aligned} T(k+1) &= 4T(\lfloor \tfrac{k+1}{2} \rfloor) + 7 && \text{by the definition} \\ &< 4(7(\lfloor \tfrac{k+1}{2} \rfloor)^2 - 3) + 7 && \text{by IH since } \lfloor \tfrac{k+1}{2} \rfloor \leq \tfrac{k+1}{2} \leq k \text{ when } k \geq 1 \\ &\leq 4(7(\tfrac{k+1}{2})^2 - 3) + 7 && \text{since } \lfloor \tfrac{k+1}{2} \rfloor \leq \tfrac{k+1}{2} \\ &= 7(k+1)^2 - 12 + 7 && \\ &< 7(k+1)^2 - 3 && \text{which is what we needed to show} \end{aligned}$$