

Algorithms

- Consider an array a_1, a_2, \dots, a_n of numbers, and suppose we want to know if a particular number b appears in the list. The following algorithm returns the *first* position i for which $a_i = b$ (if such an i exists), and returns 0 if no such i exists

```
1: LinearSearch( $a_1, a_2, \dots, a_n$  : array of reals,  $b$ :real)
2:   for  $i := 1$  to  $n$ 
3:     if  $a_i = b$  then
4:       return  $i$ 
5:     end if
6:   end for
7:   return 0    // did not find  $b$  in the array!
```

- Running time analysis: Line 1 runs once, Lines 2-6 run at most n times each, Line 7 runs at most once. Hence, the running time is $O(1) + O(n) + O(1)$ which is $O(n)$

Worst case analysis vs. Average case analysis

- Notice that the analysis on the previous page is a “worst-case” analysis. The worst case occurs when b is not in the array, and the loop executes exactly n times, and the final line (Line 7) also gets executed.
- Alternatively, we could do an average case analysis that assumes something about how likely it is for b to be in the array, and where it is likely to be if it is in the array.
- We will almost always do a worst-case analysis, but if we knew that b was definitely in the array and was equally likely to be at any location i , we could compute the *average* number of times the loop executes as:

$$\frac{1 + 2 + \dots + n}{n} = \frac{1}{n} \sum_{k=1}^n k = \frac{n(n+1)}{2n} = \frac{n+1}{2} = O(n)$$

- Hence, the average running time is also $O(n)$