Announcements

- **Midterm 1 is on Wednesday**
- Time: 7pm to 9pm
- Location: 141 Wohlers Hall
- Lecture on Wednesday is optional: exam-related Q&A
- Extra office hour on Wednesday: 11am to 12pm

Another False Proof

- "**Claim**": $\mathbf{P}(A) \cup \mathbf{P}(B) = \mathbf{P}(A \cup B)$

- "**Proof**": Let $S \in P(A) \cup P(B)$. Then $S \in P(A)$ or $S \in P(B)$

  So $S \subseteq A$ or $S \subseteq B$

  So $S \subseteq A \cup B$

  Hence $S \in \mathbf{P}(A \cup B)$

  Let $S \in \mathbf{P}(A \cup B)$

  So $S \subseteq A \cup B$

  So $\forall x,\ x \in S \rightarrow x \in A \cup B$

  $\forall x,\ x \in S \rightarrow x \in A$ or $x \in B$

  $S \subseteq A$ or $S \subseteq B$

  So $S \in \mathbf{P}(A)$ or $S \in \mathbf{P}(B)$

  So $S \in \mathbf{P}(A) \cup \mathbf{P}(B)$

Functions

- In a programming language like C or Java, we declare functions like this:

```
int f(int x) {  // int argument, int return value
  return 2*x;
}
```

- In general, a function each argument a unique return value

- The set from which arguments are drawn is called the domain
- The set from which return values are drawn is called the co-domain
- In our example, arguments and return values are both `int`s

- So, $f : \mathbf{Z} \longrightarrow \mathbf{Z}$ and is defined by $f(x) = 2x$

Functions contd.

- Two functions are equal if they have the same domain, and the same co-domain, and assign the same output value to each input value

- *Examples*: The function $g : \mathbf{N} \longrightarrow \mathbf{N}$ defined by $g(x) = 2x$ is *different* to f, because it has a different domain and co-domain

  The function $h : \mathbf{Z} \longrightarrow \mathbf{Z}$ defined by $h(x) = x + x$ is equal to f

- A function is sometimes called a "mapping" or "map", because it *maps* objects in the domain to objects in the co-domain

- A function
  - can map "many to one" e.g., $f : \mathbf{N} \longrightarrow \mathbf{N}$ defined by $f(x) = 0$
  - *cannot* map "one to many" e.g., "numbers to phones" not a function
  - *must* map everything in the domain
  - may not map to every item in the co-domain

Function or not?

- $f : \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = y$ such that $x.y = 1$
- $g : \mathbf{Z} \longrightarrow \mathbf{Z}$ defined by $g(x) = y$ such that $x + y = 0$
- $h : \mathbf{R} \longrightarrow \mathbf{R}$ defined by $f(x) = y$ such that $y^2 = x$
- $g : \mathbf{Z} \longrightarrow \mathbf{Z}$ defined by $g(x) = \lceil x \rceil$
- Two special types of functions:
- **One-to-one**: each domain item mapped to *exactly one* co-domain item
- **Onto:** maps to *every* item in the co-domain

$f : A \longrightarrow B$ is one-to-one if

$\forall x \in B, \exists y \in A, \; f(y) = x$

$f : A \longrightarrow B$ is onto if

$\forall x \in A, \forall y \in A, \; f(y) = f(x) \rightarrow x = y$

Image and Pre-Image

- The image of a function is the set of values it actually maps to:

  If $f : A \longrightarrow B$ then $Im(f) = \{ f(x) \mid x \in A \}$

- The pre-image of an object in the co-domain is the set of objects in the domain that map to it

  For $y \in B$, $f^{-1}(y) = \{ x \in A \mid f(x) = y \}$

- To recap: A function $f : A \longrightarrow B$ is
  - onto if $Im(f) = B$
  - one-to-one if $\forall y \in Im(f),\ \left| f^{-1}(y) \right| = 1$

- A function is bijective if it is both one-to-one and onto