Announcements

- Quiz 1 solutions available
- Graded quizzes will be returned next week

Sets

- A set is an unordered collection of objects

- Three ways to define a set:
  1. Describe its contents in mathematical English (e.g., the set of all integers greater than 7)

  2. List all the members of the set (e.g., { 8, 9, 10, ... })

  3. Use set-builder notation: $\{ x \in \mathbf{Z} \mid x > 8 \}$

     name of variable

     constraint(s)

     such that

- The set of all multiples of 7: $\{ x \in \mathbf{Z} : 7 \mid x \}$

Careful with set notation

- The sets $\{1, 2, 3\}$ and $\{3, 2, 1\}$ are identical
- $\{1, 2, 3, 1\}$ and $\{1, 2, 3\}$ are identical
- The empty set is written as ϕ (not $\{\ \}$)
- A set can contain objects of more than one type:
  - { cow, 6, ! }
  - $\{ a, \{ a, b \} \}$
  - $\{ \mathbf{Z}, \mathbf{Q} \}$
- The set $\{ \phi \}$ contains one object, the empty set
- The cardinality of set A, written as $|A|$, is the number of objects in A
  - Don't confuse the notation with absolute value

Subset

- **Definition**: Set A is a subset of set B if every object in A is in B

  $A \subseteq B$ if $\forall x, \; x \in A \rightarrow x \in B$

- *Examples*: $\mathbf{Z} \subseteq \mathbf{Q}$, $A \subseteq A$ for any set A

- **Claim**: For any set B, $\phi \subseteq B$

- A is a proper subset of B $(A \subset B)$ if $A \subseteq B$ and $A \neq B$

- Which of these is correct: If $A \subset B$ then

  $\exists x \in A, \; x \notin B$

  $\exists x \in B, \; x \notin A$

  $\forall x \in A, \; x \in B$

  $\{ x \in B \mid x \notin A \} \neq \phi$

Set Operations

- Intersection : $A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$
- Union : $A \cup B = \{ x \mid x \in A \text{ or } x \in B \}$
- Difference : $A - B = \{ x \mid x \in A \text{ and } x \notin B \}$
- Complement : $\overline{A} = \{ x \in U \mid x \notin A \}$
- *Examples*: If $A \subseteq B$ then

  $A \cap B = A$

  $A \cup B = B$

  $A - B = \phi$

Power set and Cartesian product

- The power set of set A, written as $\mathbf{P}(A)$, is the set of all subsets of A

- *Examples*: $\mathbf{P}(\{0, 1\}) = \{ \phi, \{0\}, \{1\}, \{0, 1\} \}$
  $\mathbf{P}(\phi) = \{ \phi \}$

- If A is a set such that $|A| = n$, then $|\mathbf{P}(A)| = 2^n$
  $\mathbf{P}(A)$ is sometimes written as 2^A

- The Cartesian product of set A and B, written $A \times B$, is the set of all ordered pairs (x, y) where $x \in A$ and $y \in B$
  $A \times B = \{ (x, y) \mid x \in A \text{ and } y \in B \}$
  $|A \times B| = |A| . |B|$

- In general, $A \times B \neq B \times A$