Announcements

- HW 1 released last Friday, due this Friday

  http://www.cs.uiuc.edu/class/sp10/cs173

Logical Equivalences

- Definition: Two propositions p and q are logically equivalent if they have the same truth-tables (notation: $p \equiv q$)

- *Examples*: $(p \to q) \equiv (\neg p \vee q)$
  $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$
  $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$

- Try one: Show that $p \vee (q \wedge r)$ is logically equivalent to $(p \vee q) \wedge (p \vee r)$

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	*T*	*T*	*T*	*T*
T	*T*	*F*	*T*	*T*
T	*F*	*T*	*T*	*T*
T	*F*	*F*	*T*	*T*
F	*T*	*T*	*T*	*T*
F	*T*	*F*	*F*	*F*
F	*F*	*T*	*F*	*F*
F	*F*	*F*	*F*	*F*

Negating Propositions

- DeMorgan's Laws and the fact that $\neg(\neg(p)) \equiv p$ are useful

- Another useful fact: $\neg\,(p \rightarrow q)$

  $\equiv \neg(\neg p \vee q)$ by the earlier equivalence

  $\equiv (\neg(\neg p)) \;\wedge\; (\neg q)$ by DeMorgan's Law

  $\equiv p \wedge (\neg q)$

- *Example*: Negate: "If it rains then the road gets slippery"
  - "if p then q" is the same as $p \rightarrow q$
  - It rains and the road does not get slippery

- Negate: "If it rains then the road gets slippery and visibility drops"

- It rains and (the road does not get slippery or the visibility does not drop)

Tautologies and Contradictions

- A tautology is a proposition in which all truth-table entries are true (T)
  - Example: $p \vee (\neg p)$
- A contradiction is a proposition in which all truth-table entries are false (F)
  - Example: $p \wedge (\neg p)$
- $p \equiv q$ when $p \leftrightarrow q$ is a tautology
- Classify as tautology, contradiction, or neither:
- $(p \vee q) \vee (\neg q)$
- $p \vee (p \wedge p)$
- $q \wedge ((\neg q) \vee (p \wedge (\neg q)))$

Predicates and Variables

- A predicate is a statement that becomes a proposition when we substitute values for all variables.
- *Examples*: "$x^2 < 10$" or "My current grade in course x is y"
- We use the notation $P(x)$ to denote predicate P with variable x
  - $Q(x, y)$ denotes a predicate with two variables
- Given a predicate $P(x)$, we can ask two kinds of questions:
  - Is there a value for variable x for which $P(x)$ is true?
  - Is $P(x)$ true for all possible values of x?
- In both cases, an important point will be: what possible values can x take?