Announcements

- Look out for HW 1 on the web page later today

  http://www.cs.uiuc.edu/class/sp10/cs173

- Weights for exams, homeworks, etc. have been posted

Sets

- (Informal) A collection of unique items (no repeats allowed)

- *Notation*: Use capital letters for sets, lower-case letters for set members

  $x \in A$ means x is a member of set A

  $x \notin A$ means x is not a member of set A

- *Examples*:

  Z = { ..., –3, –2, –1, 0, 1, 2, 3, ... } all integers

  $\mathbf{Z}^+$ = { 1, 2, 3, ... } positive integers

  N = { 0, 1, 2, 3, ... } natural numbers (includes 0)

  Q rational numbers

  of the form p/q where $p \in \mathbf{Z}$ and $q \in \mathbf{Z}$ and $q \neq 0$

  R real numbers

  rationals, plus irrationals like π and $\sqrt{2}$

  C complex numbers

  of the form $(a + bi)$ where $a, b \in \mathbf{R}$, and $i = \sqrt{-1}$

More set notation

- Often, we will be dealing with **N** or **Z**, so make sure that you "stay within the set"
  - *Example*: If $p, q \in \mathbf{N}$ then p/q may not be a natural number
- Related point: Don't use calculus on problems that don't require it
  - *Example*: Program *A* needs $(3n - 4)^2$ steps to process *n* numbers, whereas Program *B* needs $3n - 2$ steps. Which is faster?
- Range of numbers: The range $[a, b]$ denotes *a* to *b* inclusive
  - If you don't want the endpoints, use (a, b)
  - If you don't want one of the endpoints, use $(a, b]$ or $[a, b)$
- Pairs of numbers: $\mathbf{R}^2$ denotes the set of all pairs of real numbers
  - *Example*: $(-3.0, 0.007) \in \mathbf{R}^2$
  - Order matters! $(-3.0, 0.007) \neq (0.007, -3.0)$

Exponentials and logs

- If $n \in \mathbf{Z}^+$ and $b \in \mathbf{R}$, then b^n = b multiplied by itself n times

- Here is how b^n is defined for some other values of n :

  $b^0 = 1$ for any $b \neq 0$ (0^0 is undefined)

  $b^{0.5} = \sqrt{b}$

  $b^{-1} = 1/b$

- Some rules to manipulate exponents:

$$b^x b^y = b^{x+y} \quad (b^x)^y = b^{xy} \quad (b^x)^y \neq b^{(x^y)}$$

- If $b > 1$, the "inverse" of b^x is $\log_b x$ (log of x to the base b)

  — *Note*: In this class, $\log x$ means $\log_2 x$ (base 2 because of binary)

$$\log_b(1/x) = -\log_b x \quad \log_b(xy) = \log_b x + \log_b y \quad \log_b(x^y) = y \log_b x$$

- Change of base formula: $\log_b x = \log_a x \log_b a$

Floor and ceiling

- If $x \in \mathbf{R}$, $\lfloor x \rfloor$ is the *largest* integer smaller than or equal to x
  – i.e., $\lfloor x \rfloor$ is what you get when you round x downwards
- If $x \in \mathbf{R}$, $\lceil x \rceil$ is the *smallest* integer greater than or equal to x
  – i.e., $\lceil x \rceil$ is what you get when you round x upwards
- *Examples*:

$\lfloor 0.999 \rfloor = 0$

$\lceil 1.001 \rceil = 2$

$\lfloor -0.999 \rfloor =$

$\lceil -1.001 \rceil =$

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Summations

- The sum $a_1 + a_2 + \ldots + a_n = \sum_{i=1}^{n} a_i$

- For example, suppose $a_i = 2^i$. Then $\sum_{i=1}^{n} a_i = \sum_{i=1}^{n} 2^i = 2^{n+1} - 2$

  closed form

- *Question*: What is the closed form for $\sum_{i=0}^{n} 2^i$

- A similar notation for product: $\prod_{i=1}^{n} a_i$

- *Question*: Which of these is the correct closed form for $1 + 2 + \ldots + n$

$$n^2 - 1 \qquad \sum_{i=1}^{n} i \qquad \frac{n(n+1)}{2}$$