CS 173: Discrete Mathematical Structures, Spring 2010
Honors Homework 2

Due by 5pm on Monday April 5th. Please give to Margaret or push it under the door of Margaret's office (3214 Siebel).

Your homework must be formatted using latex. Please turn in hardcopy (latex output only, don't worry about the source).

1. Consider a function $f : A \to B$ and a function $g : C \to B$, where $C \supseteq A$. We say that g is an *extension* of f if $\forall x \in A,\ f(x) = g(x)$. In other words, g behaves exactly the same way as f on all elements in the domain of f.

   Question: Let A and B be finite non-empty sets. Prove that there are $|B|^{|A|}$ *distinct* functions with domain A and co-domain B.

   Hint: Prove the result by induction on $|A|$. For the inductive step, let $|A| = k$ and let $C \supseteq A$ such that $|C| = k + 1$. Notice that any function $g : C \to B$ is the extension of some function $f : A \to B$.

2. The Pell sequence is defined to be

$$\begin{aligned} P_1 &= 1 \\ P_2 &= 2 \\ P_n &= 2P_{n-1} + P_{n-2} \end{aligned}$$

   Prove that $P_{n+1}P_{n-1} - (P_n)^2 = (-1)^n$ for every $n \geq 2$.

3. Consider the following variation of the two-player game Nim. The game starts with a *single* pile of n matches, and each player takes turns to pick up 1, 2, or 3 matches. The player who picks up the last match *loses*. Prove that $\forall n \geq 1$, Player 1 can force a win if $n \not\equiv 1 \pmod 4$, and Player 2 can force a win otherwise.