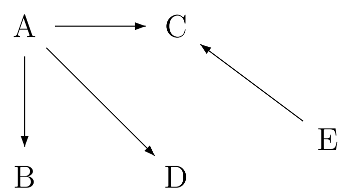
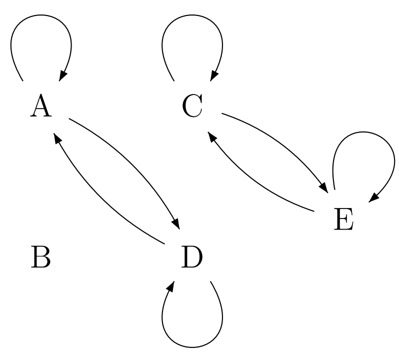
# CS 173, Quiz 3, Wednesday 28 April, 9am

# Quiz 3: Solutions

1. (6 points) Check all boxes which correctly characterize this relation, leaving the other boxes blank. (If you change your answer, make it very clear when you've meant to uncheck a box.)




| | | | |
|---|---|---|---|
| **Reflexive**: | [ ] | **Irreflexive**: | [X] |
| **Symmetric**: | [ ] | **Antisymmetric**: | [X] |
| **Transitive**: | [X] | | |




| | | | |
|---|---|---|---|
| **Reflexive**: | [ ] | **Irreflexive**: | [ ] |
| **Symmetric**: | [X] | **Antisymmetric**: | [ ] |
| **Transitive**: | [X] | | |

2. (3 points) State the "Handshaking Theorem" for graphs.

   For any graph, the sum of the degrees of all vertices is equal to twice the number of edges.

3. (3 points) How many 4-digit numbers are greater than or equal to 1000 and also a multiple of 5?

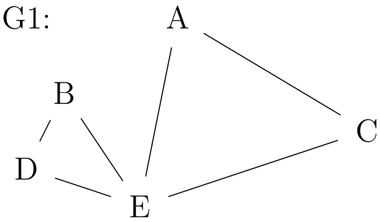
   **Solution**: $9 \times 10 \times 10 \times 2$, which is the number of 4-digit numbers where the first digit is non-zero and the last digit is 0 or 5.

   You and your $n$ friends form a *social graph*: each person is a vertex, and there is an undirected edge between every pair of people who are friends. If two more people become your friends, what is the *maximum* number of edges that might be added to your social graph?

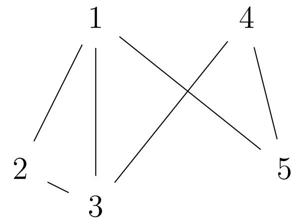
   **Solution**: $2 + 2n + 1$: Each new friend now has an edge to you (two edges), they could both also have had edges to each of your $n$ previous friends ($2n$ edges), and they could themselves have been friends (one edge).

4. (2 points) Prove that the following two graphs are not isomorphic.

G1:

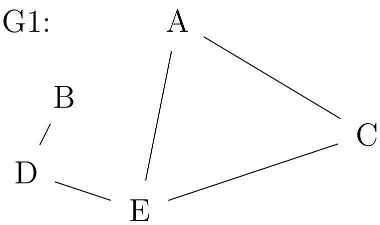



G2:

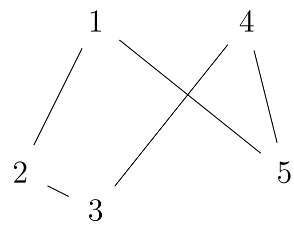



**Solution**: G1 has a vertex of degree 4 whereas G2 does not.

G1:




G2:




**Solution**: G1 has a vertex of degree 1, whereas G2 does not.

5. (2 points) How many edges are in the complete bipartite graph $K_{11,6}$? How many edges are in the complete bipartite graph $K_{5,7}$?

**Solution**: $11 \times 6$ and $5 \times 7$ respectively.

6. (4 points) Define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows:

$$(x, y) \sim (p, q) \quad \text{iff} \quad xy \equiv pq \pmod 5$$

(a) List four elements of $[(2, 3)]$.
**Solution**: $(2, 3), (3, 2), (6, 1), (1, 6)$

(b) Is $[(2, 3)]$ finite or infinite? Briefly justify your answer.
**Solution**: Infinite, because it contains all pairs of the form $(5m + 1, 1)$ where $m \in \mathbb{Z}$.

Let $A = \{(x, x + 1) \mid x \in \mathbb{N}\}$. Define the equivalence relation $\sim$ on $A$ as follows:

$$(x, x + 1) \sim (y, y + 1) \quad \text{iff} \quad x \equiv y \pmod 3$$

(a) List four elements of $[(2, 3)]$.
**Solution**: $(2, 3), (5, 6), (8, 9), (11, 12)$

(b) Is $[(2, 3)]$ finite or infinite? Briefly justify your answer.
**Solution**: Infinite, because it contains all pairs of the form $(3m + 2, 3m + 3)$, where $m \in \mathbb{N}$.