CS 173: Discrete Structures, Spring 2010
Quiz 2 (Wednesday 17 March)

NAME:

NETID:

DISCUSSION DAY/TIME:

This quiz has 2 pages containing 4 questions, totalling 20 points. You have 20 minutes to finish. Showing your work may increase partial credit in case of mistakes.

1. (4 points) Mark the following claims as "true" or "false":

   (a) There is a one-to-one function $f : \{0,1,2\} \to \{a,b\}$
   ALTERNATE: There is an onto function $f : \{a,b\} \to \{0,1,2\}$

   (b) If $f : \mathbb{Z}^+ \to [0,1]$ is defined as $f(n) = \frac{1}{n}$, then f is onto.
   ALTERNATE: If $f : \mathbb{Z}^+ \to [0,1]$ is defined as $f(n) = \frac{1}{n}$, then f is one-to-one.

   (c) For functions f, g and h, if f is $O(g)$ and g is $O(h)$ then f is $O(h)$.
   ALTERNATE: For functions f, g and h, if f is $\Omega(g)$ and g is $\Omega(h)$ then f is $\Omega(h)$.

   (d) For any $A \subseteq \mathbb{Z}$, there is a $y \in \mathbb{Z}$ such that for all $x \in A$, $x \geq y$.

2. (3 points) Suppose that $f : A \to B$ is a function. Define what it means for f to be one-to-one. Be specific and precise; do not use words like "unique."

   ALTERNATE: replace one-to-one with onto.

3. (8 points) Check all the boxes which correctly characterize each function, leaving the other boxes blank. (ALTERNATES: see end.)

$f : \mathbb{N} \to \mathbb{N}$ such that $f(n) = 3n^3 + 7(n^2 - n)$	$O(n^3)$: ☐	$\Omega(n^2)$: ☐
	$O(2^n)$: ☐	$\Omega(n!)$: ☐
$g : \mathbb{N} \to \mathbb{N}$ such that $g(n) = 2\left(\sum_{i=0}^{n} i\right)$	**One-to-one**: ☐	**Onto**: ☐
$h : \mathbb{N}^2 \to \mathbb{Q}$ such that $h(x, y) = \frac{x}{y+1}$	**One-to-one**: ☐	**Onto**: ☐

4. (5 points) Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by $f(1) = 1$, $f(2) = 5$, and, for $n \geq 3$, $f(n) = 5f(n-1) - 6f(n-2)$. Fill in the following key pieces of a proof by induction showing that $f(n) = 3^n - 2^n$, for all integers $n \geq 1$. Spell out formulas explicitly; do not use vague references like "the claim" or "the formula."

Base case or cases:

Inductive hypothesis (i.e. what you assume at the start of the inductive step):

Goal of the inductive step (i.e. the conclusion you must reach at the end of the inductive step):

Alternate for second function in problem 3

$h : \mathbb{R} \to \mathbb{R}$ such that
$h(x) = \lfloor x \rfloor$

One-to-one: ☐ **Onto**: ☐

Alternate for third function in problem 3

$h : \mathbb{N} \to \mathbb{R}$ such that
$h(n) = n \log_2(1 + n)$

One-to-one: ☐ **Onto**: ☐