CS 173: Discrete Structures, Spring 2010
Quiz 1 Solutions

Notice that the two sections had slightly different versions of many questions, and this document contains solutions to all of them.

1. (1 point) Give the day and time when your assigned discussion section meets. State explicitly if you have switched sections recently.

   Solution: This obviously varies from person to person.

2. (4 points) Simplify or find the values for the following expression:

   Solution:

   $\log_k(k^2) = 2\log_k(k) = 2$

   $(\log_k k)^2 = 1^2 = 1$

   $\gcd(42, 15) = 3$

   $\text{lcm}(6, 21) = 42$

3. (4 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

   Solution:

   $p \wedge (p \vee q) = p$

   $(p \wedge q) \vee q = q$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

p	q	$p \wedge q$	$(p \wedge q) \vee q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

4. (4 points) If $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$, give a closed-form expression for the following summation. Show your work.

   Solution:

$$\sum_{k=0}^{n+1} 2^{k-1} = \sum_{k=-1}^{n} 2^k = \frac{1}{2} + \sum_{k=0}^{n} 2^k = \frac{1}{2} + 2^{n+1} - 1 = 2^{n+1} - \frac{1}{2}$$

$$\sum_{k=0}^{n+1} (2^k - 1) = \sum_{k=0}^{n+1} 2^k - \sum_{k=0}^{n+1} 1 = (2^{n+2} - 1) - (n+2) = 2^{n+2} - n - 3$$

5. (4 points) Label each of the following equivalences, formulas, and claims as "true" or "false."

   (a) $3 \mid -6$ **True.** $-6 = 3 \cdot (-2)$

   (b) $\sqrt{2} \in \mathbb{Q}$ **False.** $\sqrt{2}$ **is irrational.**

   (c) -3 is prime **False. Primes are defined to start at 2.**

   (d) For any real number x, $2\lfloor x \rfloor = \lfloor 2x \rfloor$ **False. Suppose** $x = 1.6$**. Then** $2\lfloor x \rfloor = 2 \cdot 1 = 2 \neq 3 = \lfloor 3.2 \rfloor = \lfloor 2x \rfloor$

   (e) $(\lfloor x \rfloor)^2 = \lfloor x^2 \rfloor$ **False. Suppose** $x = \sqrt{2}$**. Then** $(\lfloor x \rfloor)^2 = 1^2 \neq \lfloor 2 \rfloor = \lfloor x^2 \rfloor$

   (f) There are integers m, n such that $n \mid m$ and $m \mid n$ **True. E.g.** $m = n = 3$

   Solution:

6. (3 points) State the negation/contrapositive of the following statement, using logical equivalences to put it into a form where each "not" is on an individual (non-complex) proposition. Show your work and give your final answer in words (not logical shorthand).

   (Give negation.) For any theory T, if T is pitted, then T is both jeffy and harpled.

   Solution: There is a theory T such that T is pitted and T is not jeffy or T is not harpled.

   (Give contrapositive) For any theory T, if T is both jeffy and harpled, then T is also pitted.

   Solution: For any theory T, if T is not pitted, then T is not jeffy or T is not harpled.