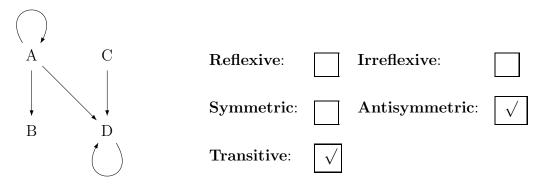
CS 173: Discrete Structures, Spring 2009 Quiz 3 Solutions

1. (5 points) Check all boxes which correctly characterize this relation, leaving the other boxes blank. (If you change your answer, make it very clear when you've meant to uncheck a box.)

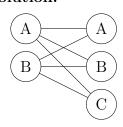


2. (2 points) Which of the following sets have the same cardinality: the integers, the rationals, the reals?

Solution: The integers and the rationals. They are both countable, whereas the reals are uncountable.

3. (3 points) Draw a picture of the graph $K_{2,3}$.

Solution:



4. (3 points) Suppose that R is a relation on a set A. Define what it means for R to be antisymmetric.

Solution: For any x and y in A, if xRy and yRx, then x = y.

- 5. (3 points) Ali is buying 15 cans of soda for the computer graphics lunch. The nearby corner shop has four kinds of soda in stock: orange, sprite, coke, and Irn-bru. He needs to decide how many sodas of each kind to get (including possibly not getting any of certain kinds). How many different choices does he have?
 - **Solution:** $\binom{18}{3}$ (which is equal to $\binom{18}{15}$). See the formulas for combinations with repetition (lecture 30). You have a list of 15 soda cans into which you must insert three dividers (to separate the four types of soda). So there are 15 + 3 positions from which you must select 3 for the dividers.
- 6. (4 points) Ellen's parents have a complex social life, so if she tries to call them on Saturday, there's only a 0.4 probability that they will answer the phone. If she tries to call on five Saturdays, what is the probability that they will answer on at least three of these days? (A formula is ok. You don't need to try to reduce it to a single number.)

Solution: Use the Bernouilli trials formula to figure out the probability for exactly 3, exactly 4, and exactly 5 successes. Then add them up (simplifying the expression for 5 successes).

$$\binom{5}{3}(0.4)^3(0.6)^2 + \binom{5}{4}(0.4)^4(0.6) + (0.4)^5$$