CS 173: Discrete Structures, Fall 2009
Quiz 2 (Wednesday 21 September)

NAME:

NETID:

This quiz has 2 pages containing 4 questions, totalling 20 points. You have 20 minutes to finish. Showing your work may increase partial credit in case of mistakes.

1. (3 points) Mark the following claims as "true" or "false":

   (a) If a function $f : \mathbb{R} \to \mathbb{R}$ is increasing, then f is one-to-one.

   (b) For any real numbers a and b, with $a > 1$ and $b > 1$, $\log_a(x)$ is $O(\log_b(x))$.

   (c) For any set A, $\{\emptyset\} \in \mathbb{P}(A)$.

2. (8 points) Check all the boxes which correctly characterize each function, leaving the other boxes blank.

   $f : \mathbb{N} \to \mathbb{N}$ such that $f(n) = 3n^3 - 7n^2 + 37$

   $O(n^3)$: ☐ $\Omega(7n^3)$: ☐

   $O(2^n)$: ☐ $\Omega(n!)$: ☐

   $g : \mathbb{N} \to \mathbb{N}$ such that $g(n) = n!$

   One-to-one: ☐ **Onto**: ☐

   $h : \mathbb{Z} \to \mathbb{Z}$ such that $h(n) = |n|$

   One-to-one: ☐ **Onto**: ☐

3. (4 points) Suppose that f and g are functions whose inputs and outputs are positive real numbers. Define precisely what it means for f to be $O(g)$. Your definition must not use Θ, Ω, or slopes/derivatives.

4. (5 points) Suppose you are proving that $\sum_{k=1}^{n} k(k+3) = \frac{n(n+1)(n+5)}{3}$ for all positive integers n. Supply the following key pieces of a proof by induction on n. Spell out the details explicitly; don't use vague references like "the claim" or "the formula."

Base case or cases:

Inductive hypothesis (i.e. what you assume at the start of the inductive step):

Goal of the inductive step (i.e. the conclusion you must reach at the end of the inductive step):