

CS 173: Discrete Structures, Fall 2009

Quiz 1 Solutions

1. (1 point) Give the day and time when your assigned discussion section meets. State explicitly if you have switched sections recently.

Solution: The right answer depends on what section you are in. Almost everyone got this one right, which is what I had expected.

2. (4 points) Compute the following quantities.

(a) $\text{lcm}(21, 35) =$ **Solution:** 105. I.e. $3 \times 5 \times 7$.

(b) $2^{\lfloor -2.8 \rfloor} =$ **Solution:** $2^{-3} = \frac{1}{8}$

3. (4 points) Give a closed-form expression for the following summation. Show your work.

$$\sum_{k=-2}^n \frac{1}{2^k} =$$

Solution:

$$\begin{aligned} \sum_{k=-2}^n \frac{1}{2^k} &= \frac{1}{2^{-2}} + \frac{1}{2^{-1}} + \sum_{k=0}^n \frac{1}{2^k} \\ &= 8 + 2 + \left(2 - \frac{1}{2^n}\right) \\ &= 8 - \frac{1}{2^n} \end{aligned}$$

4. (5 points) Are the following equivalences, formulas, and claims correct? Write “yes” next to the ones that work for all input values. Write “no” next to the ones that fail in some cases.

(a) $3 \mid 0$ **Solution:** Yes. 0 is a multiple of any integer (except that some authors exclude $0 \mid 0$).

(b) $0 \in \mathbb{N}$ **Solution:** Yes. That’s the convention we are using in this class.

(c) For any real number x , $\lfloor x \rfloor < \lceil x \rceil$. **Solution:** No. The two are equal if x is an integer.

(d) -3 is prime. **Solution:** No. The definition of prime excludes integers smaller than 2.

(e) $-13 \mid -26$ **Solution:** Yes. $-26 = -13 \cdot 2$.

5. (3 points) State the contrapositive of the following statement, using logical equivalences to put it into a form where each “not” is on an individual (non-complex) proposition. Show your work.

If John is a muggle and Ellen is a Wizard, then the crystal is not blue.

Solution: First, we swap the roles of hypothesis and conclusion, negating both of them, to get:

If the crystal is blue, then it’s not the case that (John is a muggle and Ellen is a Wizard).

Then simplify the negation to get

If the crystal is blue, then John is not a muggle or Ellen is not a Wizard.

If you felt sure that muggle and wizard were opposites (which may apply to some of you but not others), you might also have written

If the crystal is blue, then John is a wizard or Ellen is a muggle.

6. (3 points) Complete the following definition, using precise mathematical English and/or notation. Your definition must not use the mod, remainder, or similar operators.

If x and y are integers, $x \mid y$ if and only if . . .

Solution: $y = kx$ for some integer k .