CS 173, Fall 2009
Midterm 1, 30 September 2009

INSTRUCTIONS (read carefully)

- Fill in your name, netid, and discussion section time below. Also write your netid on the other pages (in case they get separated).

NAME:

NETID:

DISCUSSION DAY/TIME:

- There are 6 problems, on pages numbered 1 through 6. Make sure you have a complete exam. The point value of each problem is indicated next to the problem, and in the table below.
- You have 50 minutes to finish the exam.
- Points may be deducted for solutions which are correct but hard to read, hard to understand, poorly explained, or excessively complicated.
- Turn in your exam at the front. Show your ID to the proctors.
- It is wise to skim all problems and point values first, to best plan your time.
- This is a closed book exam. No notes or electronic devices of any kind are allowed. Do all work in the space provided, using the backs of sheets if necessary. See the proctor if you need more paper.
- Please bring any apparent bugs or ambiguity to the attention of the proctors.
- After the midterm is over, discuss its contents with other students **only** after verifying that they have also taken the exam (e.g. they aren't about to take a makeup exam).

Problem	1	2	3	4	5	6	total
Possible	12	11	7	6	7	7	50
Score							

Problem 1: Short answer (12 points)

State whether each of the following claims is true or false. Justification/work is not required, but may increase partial credit if your short answer is wrong.

(a) $-6 \mid -3$

(b) For every integer x, if x is a negative prime number then $x^2 < 0$.

(c) $\lceil x \rceil < \lfloor x + 1 \rfloor$ for any real number x?

(d) For any set A, $A \in \mathbb{P}(A)$.

(e) For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime.

(f) $\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$, for any sets A and B.

Problem 2: Calculation (11 points)

Calculate the values of the following expressions. Your answer to (e) must be in closed form, i.e. not using a summation. Recall that $\mathbb{P}(A)$ is the power set of A.

(a) $|\{\{2,2,3\},\{7\},\{19,23,23\}\}| =$

(b) $\{(x,y) \in \mathbb{Z}^2 \mid 0 \leq x \leq y \leq 3\} =$

(c) $\{1,2\} \times \mathbb{P}(\{3\}) =$

(d) Suppose that A is a p-element set and B is a q-element set. Then $|A \times \mathbb{P}(B)| =$

(e) (3 points) $\sum_{k=2}^{p}(k+2) =$

Problem 3: Longer answers (7 points)

(a) (3 points) Suppose that $f : A \to B$ is a function. Briefly explain the difference between the co-domain of f and the image of f.

(b) (4 points) Write pseudocode for a function that computes the gcd of two positive integers using the Euclidean algorithm. It is not critical to use precisely the same coding conventions as in lecture/Rosen, but it must be pseudocode (not e.g. C or Java code) and easy to understand.

Problem 4: Logic and proof structure (6 points)

(a) (3 points) Give the negation of the following statement moving the negatives (e.g. "not") so that they are on individual predicates (e.g. "my bike has two wheels"). It's ok to give your answer in logic shorthand but, if you do, make sure that you have clearly indicated definitions for your predicate or set symbols (e.g. "$R(x)$ means x is red").

> For every city c, there is a road r such that if c got money from House Bill 3141, then c fixed the potholes in r.

(b) (3 points) Disprove the following claim by giving a concrete counterexample and briefly explaining why it is a counterexample.

> For any integers a, b, and any positive integer p, if $(a \bmod p) < (b \bmod p)$, then $((a+3) \bmod p) < ((b+3) \bmod p)$.

Problem 5: Writing a proof (7 points)

Recall the following definition: given any positive integer m, the integers a and b are *congruent modulo* m (in shorthand: $a \equiv b \pmod{m}$) if and only if there is an integer k such that $a = b + km$.

Prove that, for any integers p, q, s, and t and any positive integer m,

$$\text{If } p \equiv q \pmod{m} \text{ and } s \equiv t \pmod{m}, \text{ then } ps \equiv qt \pmod{m}.$$

Prove this directly using the above definition, together with high school algebra. Do not use other facts about modular arithmetic proved in class or in the book. Make sure your steps are in logical order.

Problem 6: Proof by contradiction (7 points)

Prove the following claim using proof by contradiction. You **must** use this proof method and the following definition of "multiple": n is a multiple of m if and only if $n = km$ for some integer k. Work directly from this definition using algebra; do not use other facts from class or the textbook about divisibility. You can, however, assume that we know which small integers are multiples of one another, e.g. 13 is not a multiple of 4.

> For any natural numbers p and q, if p and q are multiples of 3, then the equation $pn + qm = 5$ has no integer solution. (That is, it is not possible to find integers n and m which make this equation true.)